



Multimode circular integrated optical microresonators

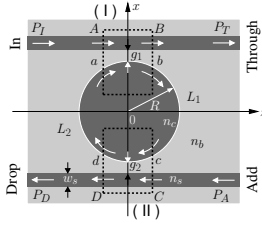
Coupled mode theory modeling



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Microresonators

- Due to their superior selectivity, compactness, and possibility of dense integration, optical microresonators (MRs) become attractive for applications as wavelength add/drop filters on photonic chips.



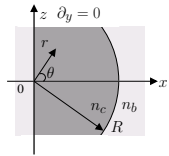
- We model a microresonator by decomposing it into two couplers, which are connected by two segments of the cavity; external connections are provided by straight waveguides.

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{B} \end{bmatrix} = \mathbf{S}^I \begin{bmatrix} \mathbf{a} \\ \mathbf{A} \end{bmatrix} \quad \begin{bmatrix} \mathbf{d} \\ \mathbf{D} \end{bmatrix} = \mathbf{S}^{II} \begin{bmatrix} \mathbf{c} \\ \mathbf{C} \end{bmatrix} \quad \begin{aligned} c_p &= b_p e^{-i\gamma_p L_1} \\ a_p &= d_p e^{-i\gamma_p L_2} \end{aligned}$$

- Given inputs $P_i^q = |A_i^q|^2$ and $P_A^q = |C_i^q|^2$, find outputs $P_T^q = |B_i^q|^2$ and $P_D^q = |D_i^q|^2$. Solve the above linear system of equations for B_i^q and D_i^q .
- Prerequisites: propagation constants γ_p , scattering matrices $\mathbf{S}^I, \mathbf{S}^{II}$
- Building blocks: analytic bend modes + coupled mode theory based model of couplers

Bent waveguides / Curved dielectric interfaces

- Field ansatz: real ω and complex $\gamma = \beta - i\alpha$

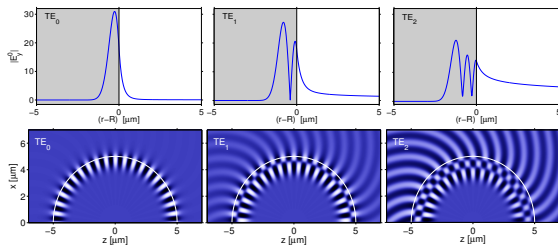


$$\begin{aligned} \mathbf{E} &= (E_r^0(r), E_\theta^0(r), E_z^0(r)) e^{i(\omega t - \gamma R\theta)} \\ \mathbf{H} &= (H_r^0(r), H_\theta^0(r), H_z^0(r)) e^{i(\omega t - \gamma R\theta)} \end{aligned}$$

- Bessel equation for $\phi = E_z^0$ (TE) or H_z^0 (TM):

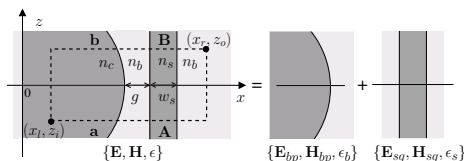
$$r^2 \frac{\partial^2 \phi}{\partial r^2} + r \frac{\partial \phi}{\partial r} + (n^2 k_0^2 r^2 - \gamma^2 R^2) \phi = 0$$

- Piecewise field ansatz + Dielectric interface conditions \rightsquigarrow Dispersion equation
- Dispersion equation + Complex order Bessel functions \rightsquigarrow Analytic bend modes



$|E_z^0|$ and real physical E_z field for a cavity with $n_c = 1.5, n_b = 1.0, R = 5 \mu\text{m}, \lambda = 1.05 \mu\text{m}$.

Multimode couplers



- Coupled Mode ansatz: C_{bp}, C_{sq} unknown amplitudes

$$\begin{bmatrix} \mathbf{E}(x, z) \\ \mathbf{H}(x, z) \end{bmatrix} = \sum_{p=1}^{N_b} C_{bp}(z) \begin{bmatrix} \mathbf{E}_{bp}(x, z) \\ \mathbf{H}_{bp}(x, z) \end{bmatrix} + \sum_{q=1}^{N_s} C_{sq}(z) \begin{bmatrix} \mathbf{E}_{sq}(x, z) \\ \mathbf{H}_{sq}(x, z) \end{bmatrix}$$

- Lorentz reciprocity theorem \rightarrow coupled mode equations

$$\mathbf{M}(z) \cdot d_z \mathbf{C}(z) = \mathbf{F}(z) \cdot \mathbf{C}(z)$$

$$M_{vp,wq} = \langle \mathbf{E}_{vp}, \mathbf{H}_{vp}; \mathbf{E}_{wq}, \mathbf{H}_{wq} \rangle = \int \mathbf{a}_z \cdot (\mathbf{E}_{vp} \times \mathbf{H}_{wq}^* + \mathbf{E}_{wq}^* \times \mathbf{H}_{vp}) dx$$

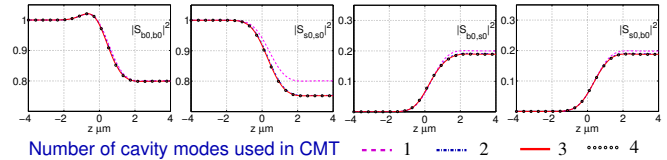
$$F_{vp,wq} = \int (\epsilon - \epsilon_v) \mathbf{E}_{vp} \cdot \mathbf{E}_{wq} dx$$

- Numerical solution $\rightsquigarrow C(z_o) = \mathbf{T} \cdot C(z_i)$
- Project the coupled fields onto the straight waveguide modes: $\mathbf{T} \rightsquigarrow \mathbf{S}$

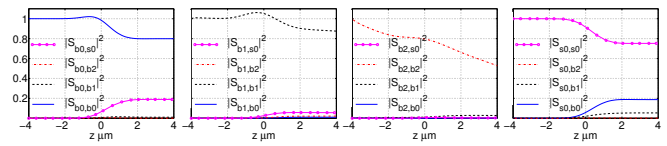
$$B_q = \left[C_{sq}(z_o) + \sum_{p=1}^{N_b} C_{bp}(z_o) \frac{\langle \mathbf{E}_{bp}, \mathbf{H}_{bp}; \mathbf{E}_{sq}, \mathbf{H}_{sq} \rangle}{\langle \mathbf{E}_{sq}, \mathbf{H}_{sq}; \mathbf{E}_{sq}, \mathbf{H}_{sq} \rangle} \right]_{z_o} e^{-i\beta_q z_o}$$

Effect of projection

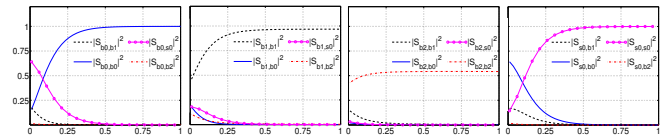
- Effect of inclusion of higher order modes on S



- Evolution of S

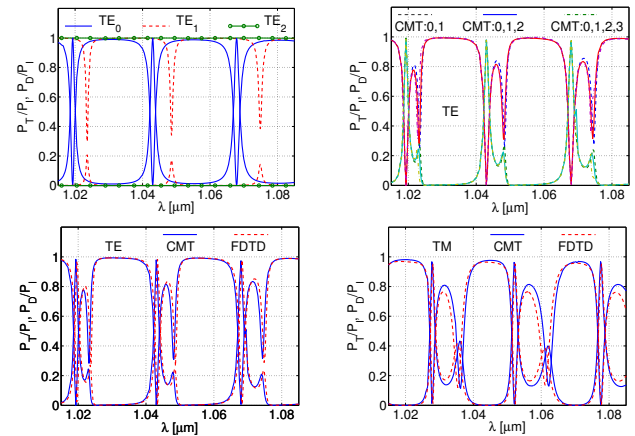


- Variation of S with the gap width

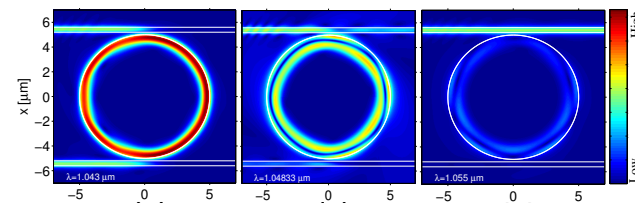


$|S_{vp,wq}|^2$ represents the power transfer from mode wq to mode vp . Parameters: $n_c = n_s = 1.5, n_b = 1.0, R = 5 \mu\text{m}, g = 0.2 \mu\text{m}, w_s = 0.4 \mu\text{m}$. Simulations with 3 cavity modes, 1 straight waveguide mode.

Microresonator spectra



Power spectra for a 2D disk MR with $n_c = n_s = 1.5, n_b = 1.0, w_s = 0.4 \mu\text{m}, R = 5 \mu\text{m}, g_1 = g_2 = 0.2 \mu\text{m}$.



TE fields $|E_y|$ for the CMT MR simulations with 3 cavity modes. From left to right: at resonance corresponding to the fundamental mode, resonance corresponding to the first order mode and off resonance.

Conclusions

- Role of the individual cavity modes can be clearly identified.
- Quite accurate results with just few dominant cavity modes.
- Low computational effort.

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