Comparison of Coupled Mode Theory and FDTD Simulations of the Coupling Between Bend and Straight Optical Waveguides

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Abstract

The analysis of the integrated optical cylindrical microresonators involves the coupling between a straight waveguide and a bend waveguide. Our (2D) variant of coupled mode theory (CMT) is based on analytically represented mode profiles. With the bend modes expressed in Cartesian coordinates, coupled mode equations are derived in a classical way and solved numerically.

Proper corrections of the propagation matrix leads to stable results even in parametric domains of compact and/or radiative structures, which seemed unsuitable for a perturbational approach due to oscillations of the matrix elements along the propagation. Comparisons with finite difference time domain (FDTD) calculations show convincing agreement.

Bend Modes

The bend fields with bend radius R in the cylindrical co-ordinate system (r, θ) are modeled by

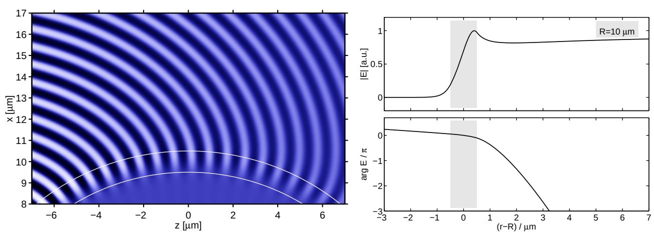
$$\mathbf{E}_b(r,\theta,t) = \mathbf{E}_b^0(r)e^{i(\omega t-\gamma R\theta)} \qquad \mathbf{H}_b(r,\theta,t) = \mathbf{H}_b^0(r)e^{i(\omega t-\gamma R\theta)}$$

Due to lossy nature of the bend modes, the propagation constant γ is complex valued.

Bessel equation:($\phi = E^0_{b,y}$ for TE waves or $H^0_{b,y}$ for TM waves)

$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} + (n^2k_0^2r^2 - \gamma^2R^2)\phi = 0$$

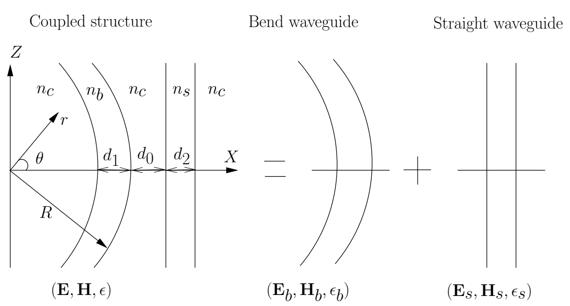
Using interface conditions, the bend modes are obtained analytically.



Physical $E_{b,y}$ bend field. Absolute value and phase of $E^0_{b,y}$.

With increasing bend radius, the bend mode confinement improves and the bend wavefront changes from a curve to a straight line.

Coupled Mode Theory



Coupled Mode Ansatz:

$$\begin{aligned}\mathbf{E}(x,z,t) &= A(z)\mathbf{E}_b(x,z,t) + B(z)\mathbf{E}_s(x,z,t)\\ \mathbf{H}(x,z,t) &= A(z)\mathbf{H}_b(x,z,t) + B(z)\mathbf{H}_s(x,z,t)\end{aligned}$$

where $A(z)$ and $B(z)$ are unknown amplitudes.

Coupled Mode Equations

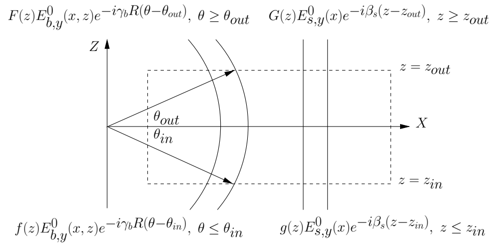
Using integral form of *Lorentz Reciprocity Theorem* we get,

$$\begin{bmatrix}\langle \mathbf{E}_b, \mathbf{H}_b^* | \mathbf{E}_b^*, \mathbf{H}_b\rangle & \langle \mathbf{E}_s, \mathbf{H}_b^* | \mathbf{E}_b^*, \mathbf{H}_s\rangle \\ \langle \mathbf{E}_b, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_b\rangle & \langle \mathbf{E}_s, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_s\rangle\end{bmatrix}\begin{bmatrix}d_zA\\ d_zB\end{bmatrix}$$

$$= -i\omega\epsilon_0\begin{bmatrix}\int \delta\epsilon_b\mathbf{E}_b\cdot\mathbf{E}_b^*dx & \int \delta\epsilon_s\mathbf{E}_s\cdot\mathbf{E}_b^*dx \\ \int \delta\epsilon_b\mathbf{E}_b\cdot\mathbf{E}_s^*dx & \int \delta\epsilon_s\mathbf{E}_s\cdot\mathbf{E}_s^*dx\end{bmatrix}\begin{bmatrix}A\\ B\end{bmatrix}.$$

where $\langle \mathbf{E}_p, \mathbf{H}_q^* | \mathbf{E}_q^* \mathbf{H}_p\rangle := \int \mathbf{a}_z \cdot (\mathbf{E}_p \times \mathbf{H}_q^* + \mathbf{E}_q^* \times \mathbf{H}_p)dx$, $\delta\epsilon_i := \epsilon - \epsilon_i$.

Solving Coupled Mode Equations



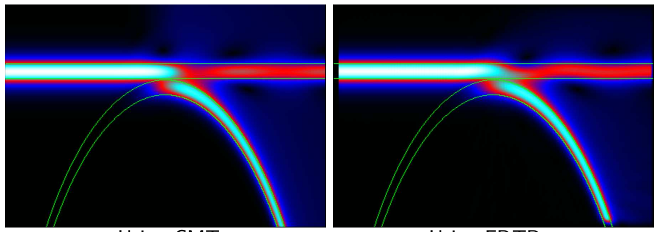
Solve coupled mode equations for propagation matrix $\mathbf{T}$ $\Longrightarrow \begin{bmatrix}A(z_{out})\\ B(z_{out})\end{bmatrix} = \begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix}\begin{bmatrix}A(z_{in})\\ B(z_{in})\end{bmatrix}$

Scattering Matrix is given by,

$$\begin{bmatrix}F(z_{out})\\ G(z_{out})\end{bmatrix} = \begin{bmatrix}T_{11}e^{i\gamma_b R(\theta_{in}-\theta_{out})} & T_{12}e^{i(\beta_s z_{in}-\gamma_b R\theta_{out})}\\ T_{21}e^{i(\gamma_b R\theta_{in}-\beta_s z_{out})} & T_{22}e^{i\beta_s(z_{in}-z_{out})}\end{bmatrix}\begin{bmatrix}f(z_{in})\\ g(z_{in})\end{bmatrix}$$

Comparison with FDTD

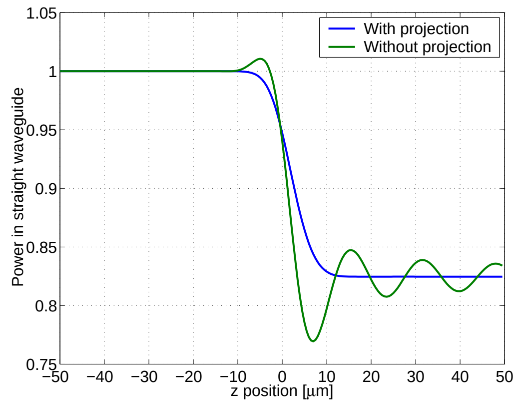
Setting : $n_b = n_s = 1.6$, $n_c = 1.45$, $d_1 = d_2 = 1\ \ \mu m$, $\lambda = 1.55\ \ \mu m$. For $d_0 = 0.5\ \mu m$ and $R = 30\ \mu m$, following is the comparison of the absolute value of $\mathbf{E}$. The CMT field is in close agreement with FDTD field.



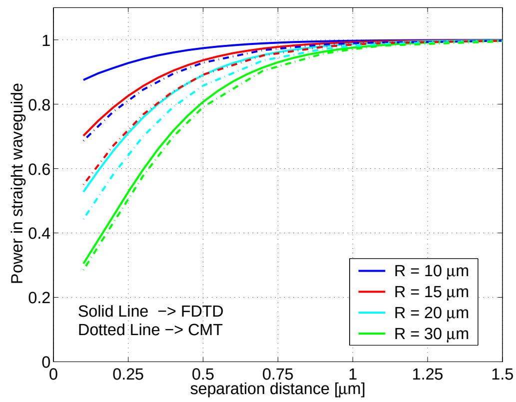
Using CMT Using FDTD

As shown in the following figure, even though the straight waveguide amplitude obtained from scattering matrix oscillates, the amplitude of the projection of coupled field on the straight waveguide field is stable.

$$G(z)|_{proj} = A(z)\frac{\langle \mathbf{E}_b, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_b\rangle}{\langle \mathbf{E}_s, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_s\rangle}e^{-i\beta z} + B(z)e^{-i\beta z}$$



As bend radius increases, CMT results agree better with FDTD results.



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