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The Monge-Ampère equation: Hamiltonian and symplectic structures, recursions, and hierarchies

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THE MONGE–AMPÈRE EQUATION: HAMILTONIAN AND SYMPLECTIC STRUCTURES, RECURSIONS, AND HIERARCHIES

P. KERSTEN, I. KRASIL′SHCHIK, AND A. VERBOVETSKY

ABSTRACT. Using methods of geometry and cohomology developed recently, we study the Monge–Ampère equation, arising as the first nontrivial equation in the associativity equations, or WDVV equations. We describe Hamiltonian and symplectic structures as well as recursion operators for this equation in its orginal form, thus treating the independent variables on an equal footing. Besides this we present nonlocal symmetries and generating functions (cosymmetries).

CONTENTS

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Introduction

In [8] we demonstrated new geometrical methods to analyse integable systems of differential equations. Although formally the theoretical constructions are not dependent on the structure of the system of equations, being of evolutionary type or more general, applications, including supersymmetric ones, were on equations of evolutionary type.

Here we apply the introduced notions to obtain generalised Hamiltonian and symplectic structures for nonevolutionary equations. To demonstrate the efficiency of these new methods of analysis of integrable systems described in [8, 9] and based on a general geometric approach to nonlinear PDE [2, 11] is the main goal. For a traditional approach to Hamiltonian formalism in integrable systems we refer the reader to [3, 12, 13, 14].

The main application is the Monge–Ampère equation, which is the first nontrivial equation in the hierarchy of Witten–Dijkgraaf–Verlinde–Verlinde equations in 2D Topological Field theory, [4] i.e.,

$$u_{yyy} - u_{xxy}^2 + u_{xxx}u_{xyy} = 0.$$

In this equation x and y play a similar role as variables; this being the reason of treating them on equal footing, and avoiding writing the equation in some more or less artificial way as an evolutionary system.

This paper is organized as follows.

In Section 1, we present the essential definitions and results needed for applications paying main attention to the computational aspects rather than to theoretical ones. All the proofs can be found in [2, 8, 9, 11]. In Section 2, the results for Monge–Ampère equation are described. The bi-Hamiltonian structure has been described by casting the (derivative of the) equation as an evolution system of hydrodynamic type, and by description as a degenerate Lagrangian system [5, 6]. Here we shall just describe all structures for the equation itself using a more natural procedure since the equation origin is such that there does not exist a distinction between the two independent variables x and y. Hamiltonian and symplectic structures are considered here in a more general way, as operators sending generating functions (cosymmetries) to symmetries and vice versa. Since the results are quite massive, the report results in a quite extensive product and computational details are presented in the Appendix.

1. Description of the computational scheme

Although application to Monge–Ampère equation is a main goal for this paper we prefer to point out the geometrical ideas for systems of equations in a more or less general setting. We deal with a system of partial differential equations of the form

$$F(x, y, v, \dots, v^j_{k,l}, \dots) = 0, \tag{1}$$

where both $F = (F^1, \dots, F^m)$ and $v = (v^1, \dots, v^m)$ are vector-functions and $v^j_{k,l} = \partial^{k+l} v^j / \partial x^k \partial y^l$, x and y being the independent variables. We choose a system of internal coordinates for the infinite prolongation of $\mathcal{E}$ (1), denoted by v_α, $\alpha \in A$, A being a set of (multi)indices. By writing $\sum_\alpha$ in the sequel we mean summation over all internal coordinates. We shall assume that the internal coordinates form a subset in the set of all coordinates $\{v^j_{k,l}\}$. Thus, any v_α is of the form $v^j_{k,l}$ for some particular k, l and j denoted by k_α, l_α and j_α, respectively. We shall also assume that the independent variables $v^1, \dots, v^m$ belong to the set of internal coordinates.

The space $\mathcal{E}$ is a subspace of the manifold of infinite jets $J^\infty(\pi)$, π being the trivial bundle $\mathbb{R}^m \times \mathbb{R}^2 \to \mathbb{R}^2$. Two basic operators related on $J^\infty(\pi)$,

$$D_x = \frac{\partial}{\partial x} + \sum_{j,k,l} v^j_{k+1,l} \frac{\partial}{\partial v^j_{k,l}}, \quad D_y = \frac{\partial}{\partial y} + \sum_{j,k,l} v^j_{k,l+1} \frac{\partial}{\partial v^j_{k,l}}$$

(total derivatives) can be restricted to $\mathcal{E}$ (i.e., expressed in terms of internal coordinates) and these restrictions will be denoted by the same symbols D_x and D_y, since only these operators will be used in the sequel.

Remark 1. Note that the above expressions for total derivatives contain infinite number of terms. To make the action of these operators (as well as of similar operators introduced below) well defined, we introduce the space $\mathcal{F}(\mathcal{E})$ of functions smoothly depending on x, y and a *finite number* of variables v_α. The operators D_x and D_y act in this space. Similarly, we shall consider the spaces $\mathcal{F}^m(\mathcal{E})$ of vector-functions of length m that depend on x, y and v_α in the same way.

1.1. **Symmetries.** A *symmetry* of equation (1) is a vector field

$$S = \sum_\alpha S^\alpha \frac{\partial}{\partial v_\alpha}, \quad S^\alpha \in \mathcal{F}(\mathcal{E}),$$

such that

$$[S, D_x] = [S, D_y] = 0.$$

Any symmetry is of the form

$$\partial_f = \sum_\alpha D_x^{k_\alpha} D_y^{l_\alpha} (f^{j_\alpha}) \frac{\partial}{\partial v_\alpha}, \tag{2}$$

where the vector-function $f = (f^1, \dots, f^m) \in \mathcal{F}^m(\mathcal{E})$ satisfies the system of equations

$$\sum_{k,l,j} \frac{\partial F^s}{\partial v^j_{k,l}} D_x^k D_y^l (f^j) = 0, \qquad s = 1, \dots, m. \tag{3}$$

The operator at the right-hand side of (3) is called the *linearization* of F and is denoted by $\ell_\mathcal{E}$. Thus, equation (3) acquires the form

$$\ell_\mathcal{E}(f) = 0. \tag{4}$$

In the sequel, we shall assume that the equation at hand satisfies the assumption of the *Vinogradov Two-Line Theorem* [15], i.e., the *compatibility complex* for the operator $\ell_\mathcal{E}$ is of length 2, see [11].

There exists a one-to-one correspondence between symmetries (2) and the corresponding functions $f \in \mathcal{F}^m(\mathcal{E})$, hence we shall identify symmetries with such functions and use the term 'symmetry' for any function that satisfies (4).

1.2. **Conservation laws and generating functions.** A *conservation law* of system (1) is a pair $\Omega = (X, Y)$, X, $Y \in \mathcal{F}(\mathcal{E})$, such that

$$D_y(X) = D_x(Y). \tag{5}$$

The function X is called the *density* of Ω. A conservation law is called *trivial* if $X = D_x(P)$, $Y = D_y(P)$ for some function $P \in \mathcal{F}(\mathcal{E})$.

Remark 2. Strictly speaking, a conservation law is the cohomological class of the form $\Omega = X\,dx + Y\,dy$ in the *horizontal de Rham complex* of $\mathcal{E}$, [2], and Ω itself is just a representative of this class.

To any conservation law there corresponds its *generating function* defined by

$$g_\Omega = \frac{\delta X}{\delta v} = \left(\frac{\delta X}{\delta v^1}, \dots, \frac{\delta X}{\delta v^m}\right),$$

where

$$\frac{\delta}{\delta v^j} = \sum_{j_\alpha = j} (-D_x)^{k_\alpha} \circ (-D_y)^{l_\alpha} \circ \frac{\partial}{\partial v_\alpha}$$

is the *variational derivative* with respect to v^j. Generating functions of conservation laws satisfy the system of equations

$$\ell^*_{\mathcal{E}}(g) = 0, \tag{6}$$

or

$$\sum_{k,l,j} (-D_x)^k (-D_y)^l \left(\frac{\partial F^j}{\partial v^s_{k,l}} g^j\right) = 0, \qquad s = 1, \dots, m, \tag{7}$$

where $\ell^*_{\mathcal{E}}$ is *adjoint* to the operator $\ell_{\mathcal{E}}$. Any conservation law is uniquely determined by its generating function and, in particular, Ω is trivial if and only if $g_\Omega = 0$. Note that equation (7) may possess solutions that do not correspond to any conservation law of (1).

Remark 3. Generating functions are also called *cosymmetries* [1] or *conserved covariants* [7].

1.3. **Nonlocal variables.** Let us introduce a set of variables $w^1, \dots, w^j, \dots$ satisfying the equations

$$w^j_x = A^j(x, y, \dots, v_\alpha, \dots, w^\beta, \dots), \quad w^j_y = B^j(x, y, \dots, v_\alpha, \dots, w^\beta, \dots), \tag{8}$$

that are compatible modulo equation (1), where A^j, B^j are some smooth functions depending on a finite number of arguments. Consider the operators

$$\widetilde{D}_x = D_x + \sum_j A^j \frac{\partial}{\partial w^j}, \quad \widetilde{D}_y = D_y + \sum_j B^j \frac{\partial}{\partial w^j}.$$

Due to the compatibility conditions, one has

$$[\widetilde{D}_x, \widetilde{D}_y] = 0. \tag{9}$$

The variables w^j are called *nonlocal*.

Using the operators $\widetilde{D}_x$, $\widetilde{D}_y$ instead of D_x and D_y in formulas (3), (5), and (7), we can introduce the notions of *nonlocal symmetries*, *nonlocal conservation laws*, and *nonlocal generating functions* depending on the new variables w^j. We shall denote the spaces of such symmetries and generating functions by $\mathbf{sym}(\mathcal{E})$ and $\mathbf{gf}(\mathcal{E})$, respectively.

Remark 4. An invariant geometric way to introduce nonlocal variables is based on the notion of *covering*, see [2, 9, 10, 11].

Remark 5. We shall exploit this notion of nonlocality in the construction of the socalled *vector-* and *form-valued* nonlocalities in the next section.

1.4. **The ℓ- and ℓ^*-extensions.** Consider an r-dimensional vector space W and the trivial fiber bundle $\tau_W\colon W\times\mathcal{E}\to\mathcal{E}$. Then the bundle of *horizontal infinite jets* $\tau_\infty\colon J_h^\infty(\tau_W)\to\mathcal{E}$ is defined. Local coordinates along the fiber of τ_∞ are $w_{k,l}^\alpha$, $k,l\geq 0$ while the total derivatives are expressed by

$$\tilde{D}_x = D_x + \sum_{k,l,\alpha} w^\alpha_{k+1,l}\frac{\partial}{\partial w^\alpha_{k,l}},\qquad \tilde{D}_y = D_y + \sum_{k,l,\alpha} w^\alpha_{k,l+1}\frac{\partial}{\partial w^\alpha_{k,l}}.$$

Obviously, $w^\alpha_{k,l}$ may be understood as nonlocal variables over $\mathcal{E}$.

Now, to any $r\times r'$-matrix differential operator $\Delta=\|\sum_{i,j}a_{ij}^{\alpha\beta}D_x^iD_y^j\|$ one can put into correspondence the subset $\mathcal{E}_\Delta\subset J_h^\infty(\tau_W)$ defined by the relations

$$\tilde{D}_x^k\tilde{D}_y^l\left(\sum_{i,j,\beta}a_{ij}^{\alpha\beta}w_{i,j}^\beta\right)=0,\qquad \alpha=1,\dots,r',\quad k,l\geq 0. \tag{10}$$

The operators $\tilde{D}_x$ and $\tilde{D}_y$ can be restricted to $\mathcal{E}_\Delta$ and, if Δ is regular, $\mathcal{E}_\Delta$ is equivalent to (1) augmented by the system

$$\sum_{i,j,\beta}a_{ij}^{\alpha\beta}\frac{\partial^{i+j}w^\beta}{\partial x^i\partial y^j}=0,\qquad \alpha=1,\dots r'. \tag{11}$$

This augmented system is called the Δ-*extension* of $\mathcal{E}$. For more details see [8]. Equations (10) can be used to introduce *internal coordinates* w_β on the space of Δ-extension.

Remark 6. Similar to equations themselves, their Δ-extensions can also be supplied with nonlocal variables (see Subsection 1.3). Variables of this type will be indicated by the 'bar' symbol: $\bar{w}_\beta$, etc.

There are two canonical ways to extend the initial system (1). The first one, the ℓ-*extension*, is related to the operator $\Delta=\ell_\mathcal{E}$. Let us denote internal coordinates on this extension by Ω_α. Then (11) will take the form

$$\sum_{k,l,j}\frac{\partial F^s}{\partial v^j_{k,l}}\frac{\partial^{k+l}\Omega^j}{\partial x^k\partial y^l}=0,\qquad s=1,\dots,m.$$

In a dual way, we construct the ℓ^*-*extension* taking the operator $\ell^*_\mathcal{E}$ for Δ. In this case, (11) is of the form

$$\sum_{k,l,j}(-D_x)^k(-D_y)^l\left(\frac{\partial F^j}{\partial v^s_{k,l}}\Pi^j\right)=0,\qquad s=1,\dots,m,$$

where Π^j are coordinates in $\mathcal{E}_\Delta$, $\Delta=\ell^*_\mathcal{E}$.

Remark 7. We shall consider the variables Ω_α and Π_α to be *odd*, see motivations in [8].

Associating operators to functions on Δ-extensions. Let $\mathcal{F}^m(\mathcal{E})$ be the space of vector functions of length m (see Remark 1). Let $a=(a_1,\dots,a_m)$, $a_i=\sum_\beta a_i^\beta w_\beta$, $a_i^\beta\in\mathcal{F}(\mathcal{E})$, be a linear in w vector-function. Then, using general properties of the horizontal jets (see above), we put into correspondence to this function a differential operator $\Delta_a=\|\Delta_a^{ij}\|\colon\mathcal{F}^m(\mathcal{E})\to\mathcal{F}^m(\mathcal{E})$, where

$$\Delta_a^{ij}=\sum_{j_\beta=j}a_i^\beta D_x^{k_\beta}D_y^{l_\beta},\qquad i,j=1,\dots,m.$$

The correspondence for functions containing nonlocal variables (see Remark 6), as they occur in results, is technically a bit more complicated and, being dependent on the representation of the nonlocalities will be exposed in the next section.

Below we shall use the notation $\mathcal{L}^m(\Delta)$ for the spaces of vector-functions linear in w and $\bar{w}$.

We can now formulate the basic property of Δ-coverings. Consider spaces of vector-valued functions P, Q, P', and Q' over $\mathcal{E}$. Assume that some operators in total derivatives $\Delta\colon P \to P'$ and $\nabla\colon Q \to Q'$ and we want to find operators V such that the diagram

$$\begin{array}{ccc} P & \xrightarrow{\ \Delta\ } & P' \\ {\scriptstyle V}\big\downarrow & & \big\downarrow{\scriptstyle V'} \\ Q & \xrightarrow{\ \nabla\ } & Q' \end{array} \tag{12}$$

is commutative for some V', both V and V' being differential operators in total derivatives.

Theorem 1. *The space of the operators V satisfying* (12) *modulo trivial solutions of the form $V = \square \circ \Delta$ is in one-to-one correspondence with w-linear solutions of the eqution*

$$\tilde{\nabla}(W) = 0,$$

where $\tilde{\nabla}$ is the lifting of the operator ∇ to the Δ-covering by substituting total derivatives D_x and D_y with $\tilde{D}_x$ and $\tilde{D}_y$, respectively.

This theorem will be extensively used in Subsections 1.5–1.8.

1.5. **Recursion operators for symmetries.** Let $R \in \mathcal{L}^m(\ell_{\mathcal{E}})$ be a function that satisfies the equation

$$\tilde{\ell}_{\mathcal{E}}(R) = \sum_{j,k,l} \frac{\partial F^s}{\partial v^j_{k,l}} \tilde{D}^k_x \tilde{D}^l_y (R^j) = 0, \qquad s = 1, \dots, m.$$

Then the corresponding operator Δ_R maps $\mathbf{sym}(\mathcal{E})$ to $\mathbf{sym}(\mathcal{E})$ and thus is a recursion operator for (nonlocal) symmetries of $\mathcal{E}$.

1.6. **Recursion operators for generating functions.** Let $L \in \mathcal{L}^m(\ell^*_{\mathcal{E}})$ be a function that satisfies the equation

$$\tilde{\ell}^*_{\mathcal{E}}(L) = \sum_{j,k,l} (-\tilde{D}_x)^k (-\tilde{D}_y)^l \left(\frac{\partial F^j}{\partial v^s_{k,l}} L^j \right) = 0, \qquad s = 1, \dots, m,$$

Then the corresponding operator Δ_L maps $\mathbf{gf}(\mathcal{E})$ to $\mathbf{gf}(\mathcal{E})$ and thus is a recursion operator for (nonlocal) generating functions of $\mathcal{E}$ (or *adjoint recursion operator* [1]).

1.7. **Hamiltonian structures.** Let $K \in \mathcal{L}^m(\ell^*_{\mathcal{E}})$ be a function that satisfies the equation

$$\tilde{\ell}_F(K) = \sum_{j,k,l} \frac{\partial F^s}{\partial v^j_{k,l}} \tilde{D}^k_x \tilde{D}^l_y (K^j) = 0, \qquad s = 1, \dots, m.$$

Then the corresponding operator Δ_K maps $\mathbf{gf}(\mathcal{E})$ to $\mathbf{sym}(\mathcal{E})$. We call such maps *pre-Hamiltonian structures or generalized Hamiltonian* (they are also known as *Noether operators* [7]).

1.8. **Symplectic structures.** Let $J \in \mathcal{L}^m(\ell_{\mathcal{E}})$ be a function that satisfies the equation

$$\tilde{\ell}^*_{\mathcal{E}}(J) = \sum_{j,k,l} (-\tilde{D}_x)^k (-\tilde{D}_y)^l \left(\frac{\partial F^j}{\partial v^s_{k,l}} J^j \right) = 0, \qquad s = 1, \dots, m.$$

Then the corresponding operator Δ_J maps $\mathbf{sym}(\mathcal{E})$ to $\mathbf{gf}(\mathcal{E})$ and may be called a *pre-symplectic or generalized symplectic structure* on $\mathcal{E}$ (alternatively, *inverse Noether operator* [7]).

2. The Monge–Ampère equation

In this section we shall study the Monge–Ampère equation, an equation in one dependent variable u and two independent variables x, y, given as

$$u_{yyy} - u_{xxy}^2 + u_{xxx}u_{xyy} = 0.$$

This equation is also the first nontrivial one in the hierarchy of WDVV equations.

Although by previous authors Hamiltonian structures were found by casting the Monge–Ampère equation as an *evolutionary* system, we shall here exploit the geometrical nature of our construction by just considering the equation itself and to establish all interesting structures. Moreover it turns out that there is no *local* Hamiltonian structure, while in this case there does exist a *local* symplectic structure. Although this application is similar to the ones discussed before, the structure and by consequence results are far more complicated than in previous cases. Although different in nature of results we proceed in a standard way in order to reach to the desired results. We make a suitable choice for the internal coordinate system. We choose this representation as x, y, u, $u_{i,0}$, $u_{i,1}$, $u_{i,2}$, $i \geq 0$.

2.1. Conservation laws and nonlocalities.

In order to start up the algorithmic procedure we construct Abelian coverings of equation

$$u_{yyy} - u_{xxy}^2 + u_{xxx}u_{xyy} = 0$$

So we construct conservation laws for $F = (F^x, F^y)$ satisfying

$$D_y(F^x) - D_x(F^y) = 0,$$

leading to the required nonlocalities.

We should stress here that due to the intrinsic structure of the equations conservation laws of *higher/deeper nonlocality* play a crucial role in the construction of the geometrical objects like Hamiltonian, symplectic and recursion operators for symmetries and generating functions.

The first index refers to the depth of nonlocality, while the second one here and below refers to the grading (a possible one) for Monge–Ampère equation:

$$\begin{aligned}
\deg(u) &= 0,\\
\deg(x) &= -1,\\
\deg(y) &= -4,\\
\deg(u_{12}) &= 0 + 1 + 2 \cdot 4 = 9
\end{aligned}$$

Remark 8. We shall only list here and elsewhere partial results, needed for presentation of the recursion operators and Hamiltonian and symplectic structures; for complete results we refer to the Appendix.

We obtained the following results.

At zero-th order we found: $Q_{0,7}$, $Q_{0,9}$, $Q_{0,12}, \ldots$ of the form

$$\begin{aligned}
(Q_{0,7})_x &= -u_{01}u_{40} + u_{02},\\
(Q_{0,7})_y &= -u_{01}u_{31} - u_{02}u_{30} + u_{11}u_{21},\\
(Q_{0,9})_x &= 2u_{02}u_{20} + u_{11}^2 - u_{20}^2 u_{21},\\
(Q_{0,9})_y &= 2u_{02}u_{11} - u_{12}u_{20}^2,\\
(Q_{0,12})_x &= u_{11}(u_{02} - u_{20}u_{21}),\\
(Q_{0,12})_y &= (u_{02}^2 - 2u_{11}u_{12}u_{20})/2.
\end{aligned}$$

At first order we found: $Q_{1,3}$, $Q_{1,6}$, $Q_{1,8},\dots$ of the form

$$
\begin{aligned}
(Q_{1,3})_x &= 2u_{01} - u_{20}^2,\\
(Q_{1,3})_y &= 2(Q_{0,7} + u_{01}u_{30} - u_{11}u_{20}),\\
(Q_{1,6})_x &= Q_{0,7} + u_{01}u_{30},\\
(Q_{1,6})_y &= u_{11}^2/2,\\
(Q_{1,8})_x &= Q_{0,9} - 2u_{01}u_{10}u_{40} - 4u_{01}u_{20}u_{30} + 2u_{02}u_{10},\\
(Q_{1,8})_y &= 4Q_{0,12} - 2u_{01}u_{10}u_{31} - 2u_{01}u_{11}u_{30} - 2u_{01}u_{20}u_{21} - 2u_{02}u_{10}u_{30}\\
&\quad - u_{02}u_{20}^2 + 2u_{10}u_{11}u_{21} + 2u_{11}^2u_{20}.
\end{aligned}
$$

At second order we found: $Q_{2,5}$, $Q_{2,7},\dots$ of the form

$$
\begin{aligned}
(Q_{2,5})_x &= -18Q_{1,6} - 2u_{01}u_{20} + 4u_{10}u_{11} + u_{20}^3,\\
(Q_{2,5})_y &= -3Q_{0,9} - 2u_{01}u_{11} + 4u_{02}u_{10},\\
(Q_{2,7})_x &= (-80Q_{0,7}u_{10} - 20Q_{1,8} - 20u_{01}^2 - 120u_{01}u_{10}u_{30} + 2u_{10}u_{20}^2u_{30} - u_{20}^4)/2,\\
(Q_{2,7})_y &= -40Q_{0,7}u_{01} - 10Q_{1,11} - 30u_{01}^2u_{30} + 20u_{01}u_{11}u_{20} + 10u_{01}u_{20}^2u_{30}\\
&\quad - 30u_{10}u_{11}^2 + u_{10}u_{20}^2u_{21} - 3u_{11}u_{20}^3.
\end{aligned}
$$

At third order we found: $Q_{3,4},\dots$ of the fom

$$
\begin{aligned}
(Q_{3,4})_x &= (Q_{2,5} + 3Q_{1,3}u_{20} - 4u_{01}u_{10})/3\\
(Q_{3,4})_y &= 2Q_{0,7}u_{10} + Q_{1,3}u_{11} - Q_{1,8} - 2u_{01}^2 - u_{01}u_{20}^2.
\end{aligned}
$$

A **fourth order** result is also presented in the Appendix.

We expect that this number of nonlocalities will be sufficient to find the nonlocal structures we have in mind for this application.

2.2. **Symmetries.** The next step in the computational process is the construction of symmetries. These symmetries will play an essential role in the construction of the *vector-valued* nonlocalities further on.

A symmetry Y has to satisfy the symmetry equation

$$\ell_{\mathcal{E}}(Y) = 0,$$

which for the Monge–Ampère equation results in the condition

$$(D_y)^3(Y) - 2u_{xxy}(D_x)^2(D_y)(Y) + u_{xyy}(D_x)^3(Y) + u_{xxx}(D_x)(D_y)^2(Y) = 0.$$

From this condition we obtain the following hierachy of (x, y)-independent symmetries, Y_*^0:

$$
\begin{aligned}
Y_0^0 &= 1,\\
Y_1^0 &= u_{10},\\
Y_4^0 &= u_{01},\\
Y_5^0 &= Q_{2,5} + 8u_{01}u_{10},\\
Y_8^0 &= -2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2.
\end{aligned}
$$

The numbering system here refers to the grading (a possible one) for Monge–Ampère equation.

We constructed hierarchies of symmetries Y_*^1, which are (x, y)-dependent (linear in x, y):

$$
\begin{aligned}
Y_{-4}^1 &= y,\\
Y_{-1}^1 &= x,
\end{aligned}
$$

$$
\begin{aligned}
Y^1_{0,1} &= xu_{10} - 4u,\\
Y^1_{0,2} &= yu_{01} + u,\\
Y^1_3 &= 4xu_{01} - Q_{1,3},\\
Y^1_{4,1} &= x(Q_{2,5} + 8u_{01}u_{10}) + 3(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01}).
\end{aligned}
$$

We constructed hierarchies of symmetries Y^2_*, which are quadratic in x and y:

$$
\begin{aligned}
Y^2_{-8} &= y^2,\\
Y^2_{-5} &= xy,\\
Y^2_{-2} &= x^2,\\
Y^2_{-1} &= x^2u_{10} + 4xyu_{01} - 4xu - yQ_{1,3},\\
Y^2_2 &= 2x^2u_{01} - xQ_{1,3} - u^2_{10}.
\end{aligned}
$$

We also list a number of (x, y)-dependent symmetries Y^3_*), which are cubic in x and y:

$$
\begin{aligned}
Y^3_{-3} &= x^3 - 2yu_{10},\\
Y^3_1 &= 12x^3u_{01} - 9x^2Q_{1,3} - 18xu^2_{10} + 2y(-Q_{2,5} - 8u_{01}u_{10}) + 24uu_{10}.
\end{aligned}
$$

This finishes for the moment the construction of symmetries.

2.3. **Generating functions.** The next step in the computational process is the construction of generating functions. These generating functions will play an essential role in the construction of the *form-valued* nonlocalities further on.

A generating function G has to satisfy the socalled equation for generating functions

$$\ell^*_{\mathcal{E}}(G) = 0,$$

which for the Monge–Ampère equation results in the condition on G

$$-(D_y)^3(G) + 2(D_x)^2(D_y)(u_{21}G) - (D_x)^3(u_{12}G) - (D_x)(D_y)^2(u_{30}G) = 0.$$

From this condition we obtain a nonlocal hierachy of (x, y)-independent generating functions, i.e., a hierarchy G^0_* and nonlocal (x, y)-dependent hierarchies, the first few of them being given by

$$
\begin{aligned}
G^0_0 &= 1,\\
G^0_2 &= u_{20},\\
G^0_5 &= u_{11},\\
G^0_6 &= -18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u^3_{20},\\
G^0_9 &= 2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u^2_{20}.
\end{aligned}
$$

We list a number of (x, y)-dependent generating functions G^1_*, which are linear in x and y:

$$
\begin{aligned}
G^1_{-4} &= y,\\
G^1_{-1} &= x,\\
G^1_{1,1} &= xu_{20} - 3u_{10},\\
G^1_{1,2} &= yu_{11} + u_{10},\\
G^1_4 &= 4xu_{11} + 2u_{01} + u^2_{20}.
\end{aligned}
$$

We list a number of (x, y)-dependent generating functions G^2_*, which are quadratic in x and y:

$$G^2_{-2} = 3x^2 - 2yu_{20},$$

$$G^2_0 = x^2u_{20} + 4xyu_{11} - 2xu_{10} + y(2u_{01} + u^2_{20}) - 4u,$$
$$G^2_3 = 2x^2u_{11} + x(2u_{01} + u^2_{20}) - Q_{1,3} - 2u_{10}u_{20}.$$

We list a number of (x, y)-dependent generating functions G^3_*, which are cubic in x and y:

$$G^3_{-3} = 2u_{10}y - 2u_{11}y^2 - 2u_{20}xy + x^3,$$
$$G^3_2 = (12x^3u_{11} + 9x^2(2u_{01} + u^2_{20}) + 18x(-Q_{1,3} - 2u_{10}u_{20}),$$
$$+ 2y(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u^3_{20}) + 6(+4uu_{20} + u^2_{10}))/12.$$

This finishes for the moment the construction of generating functions.

2.4. **ℓ-covering.** In order to construct recursion operators for symmetries and symplectic operators, we have to construct the ℓ-covering of the Monge–Ampère equation, given by u and odd variables Ω, Ω_x, Ω_y, Ω_{xx}, Ω_{xy}, $\Omega_{yy}, \dots$ satisfying the condition

$$\ell_{\mathcal{E}}(\Omega) = 0,$$

which amounts to

$$(D_y)^3(\Omega) - 2u_{xxy}(D_x)^2(D_y)(\Omega) + u_{xyy}(D_x)^3(\Omega) + u_{xxx}(D_x)(D_y)^2(\Omega) = 0.$$

We have to construct nonlocalities in this ℓ-covering first, the results of which are presented in the next subsection.

2.5. **Conservation laws and nonlocalities in ℓ-covering.** Here we construct *form-valued* nonlocalities, which are given by $\Omega(76)$, $\Omega(77), \dots$ They result from nontrivial form-valued conservation laws in the ℓ-covering of the Monge–Ampère equation, defined by its differential consequences:

$$D_x(F^y) - D_y(F^x) = 0.$$

We are now choosing $F = (F^x, F^y)$ in a *canonical way*, leading to the presentation

$$F^x = G\Omega_{yy} + a_y\Omega_y + a_0\Omega,$$
$$F^y = b_{yy}\Omega_{yy} + b_{xy}\Omega_{xy} + b_{xx}\Omega_{xx} + b_y\Omega_y + b_x\Omega_x + b_0\Omega,$$

where

$$\begin{aligned} b_{yy} &= -u_{30}G, \\ b_{xy} &= 2u_{21}G, \\ b_{xx} &= -u_{12}G, \\ b_y &= -D_x(b_{xy}), \\ b_x &= -D_x(b_{xx}), \\ b_0 &= -D_x(b_x), \\ a_y &= -(D_y(G) - D_x(b_{yy})), \\ a_0 &= -(D_y(a_y) - D_x(b_y)), \end{aligned}$$

while G is a solution of

$$\ell^*_{\mathcal{E}}(G) = 0,$$

which amounts to

$$-(D_y)^3(G) + 2(D_x)^2(D_y)(u_{21}G) - (D_x)^3(u_{12}G) - (D_x)(D_y)^2(u_{30}G) = 0,$$

i.e., G is a generating function.

This presentation holds for nonlocalities of depth zero; for higher nonlocalities the theoretical presentation is more complicated. This does lead to the following *form-valued* nonlocalities $\Omega(i)$, $i = 76, \ldots, 94$, defined by their x and y derivatives. Those necessary for presentation of the results are:

$$D_x\Omega(76) = \Omega_{yy} - \Omega_y u_{40} - \Omega u_{41},$$
$$D_y\Omega(76) = -\Omega_{yy}u_{30} + 2\Omega_{xy}u_{21} - \Omega_{xx}u_{12} - 2\Omega_y u_{31} + \Omega_x u_{22} - \Omega u_{32},$$

$$\begin{aligned} D_x\Omega(77) &= \Omega_{yy}u_{20} - \Omega_y(u_{20}u_{40} + u_{21} + u_{30}^2) \\ &\quad + \Omega(-u_{20}u_{41} - u_{21}u_{40} + u_{22} - 2u_{30}u_{31}), \end{aligned}$$
$$\begin{aligned} D_y\Omega(77) &= -\Omega_{yy}u_{20}u_{30} + 2\Omega_{xy}u_{20}u_{21} - \Omega_{xx}u_{12}u_{20} - 2\Omega_y(u_{20}u_{31} + u_{21}u_{30}) \\ &\quad + \Omega_x(u_{12}u_{30} + u_{20}u_{22}) + \Omega(-u_{12}u_{40} - u_{20}u_{32} - 2u_{22}u_{30}), \end{aligned}$$

$$D_x\Omega(78) = \Omega_{yy}u_{11} - \Omega_y(u_{11}u_{40} + u_{12} + u_{21}u_{30}) + \Omega(-u_{11}u_{41} - 3u_{21}u_{31})$$
$$\begin{aligned} D_y\Omega(78) &= -\Omega_{yy}u_{11}u_{30} + 2\Omega_{xy}u_{11}u_{21} - \Omega_{xx}u_{11}u_{12} - 2\Omega_y(u_{11}u_{31} + u_{21}^2) \\ &\quad + \Omega_x(u_{11}u_{22} + u_{12}u_{21}) + \Omega(-u_{11}u_{32} - u_{12}u_{31} - 2u_{21}u_{22}), \end{aligned}$$

$$D_x\Omega(83) = \Omega_y - \Omega u_{40},$$
$$D_y\Omega(83) = \Omega(76),$$

$$D_x\Omega(84) = \Omega(76) + \Omega_y u_{30} + 3\Omega u_{31},$$
$$D_y\Omega(84) = 2\Omega_y u_{21} - \Omega_x u_{12} + 2\Omega u_{22},$$

$$\begin{aligned} D_x\Omega(85) &= \Omega(77) + 3\Omega_{yy}u_{10} + \Omega_y(-3u_{10}u_{40} - 3u_{11} - 2u_{20}u_{30}) \\ &\quad + 3\Omega(-u_{10}u_{41} + u_{11}u_{40} + u_{12} - 2u_{20}u_{31}), \end{aligned}$$
$$\begin{aligned} D_y\Omega(85) &= -3\Omega_{yy}u_{10}u_{30} + 6\Omega_{xy}u_{10}u_{21} - 3\Omega_{xx}u_{10}u_{12} + 2\Omega_y(-3u_{10}u_{31} - 2u_{20}u_{21}) \\ &\quad + \Omega_x(3u_{10}u_{22} + 2u_{12}u_{20}) + \Omega(-3u_{10}u_{32} - u_{12}u_{30} - 4u_{20}u_{22}), \end{aligned}$$

$$\begin{aligned} D_x\Omega(86) &= -\Omega_{yy}u_{10} + \Omega_y(u_{10}u_{40} + 2u_{11} + u_{20}u_{30}) \\ &\quad + \Omega(u_{10}u_{41} - 2u_{11}u_{40} - 3u_{12} + 3u_{20}u_{31}), \end{aligned}$$
$$\begin{aligned} D_y\Omega(86) &= \Omega(78) + \Omega_{yy}u_{10}u_{30} - 2\Omega_{xy}u_{10}u_{21} + \Omega_{xx}u_{10}u_{12} \\ &\quad + 2\Omega_y(u_{10}u_{31} + u_{20}u_{21}) - \Omega_x(u_{10}u_{22} + u_{12}u_{20}) \\ &\quad + \Omega(u_{10}u_{32} + u_{12}u_{30} + 2u_{20}u_{22}), \end{aligned}$$

$$D_x\Omega(91) = 3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2),$$
$$D_y\Omega(91) = \Omega(77) - 3\Omega u_{12},$$

$$D_x\Omega(94) = \Omega(91) + \Omega_y u_{10} - \Omega u_{10}u_{40},$$
$$D_y\Omega(94) = 2\Omega(86) + \Omega(85).$$

This finishes the construction of the ℓ - Covering.

2.6. **Symmetries in ℓ-covering.** In the previous two subsections we estabished the ℓ-covering of the Monge–Ampère equation, which will serve for our purposes to construct the recursion operators for symmetries, by their definition being nothing but Symmetries in the ℓ-covering of the equation $\mathcal{E}$.

In order to arrive at the recursion operators for symmetries R^S we construct *form-valued* symmetries in the ℓ-covering. From this symmetry condition

$$\ell_{\mathcal{E}}(R^S) = 0$$

we obtained the following form-valued symmetries $R^S(0)$ and $R^S(1)$:

$$R^S(0) = \Omega,$$
$$R^S(1) = -\Omega(94) + \Omega(83)u_{10}.$$

It should be noted that $R^S(0)$ represents just the identity operator, and that *completely formally*

$$\begin{aligned}
\Omega(94) &= (D_x)^{-1}(\Omega(91) + \Omega_y u_{10} - \Omega u_{10}u_{40}),\\
\Omega(91) &= (D_x)^{-1}(3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)),\\
\Omega(84) &= (D_x)^{-1}(\Omega(76) + \Omega_y u_{30} + 3\Omega u_{31}),\\
\Omega(76) &= (D_x)^{-1}(\Omega_{yy} - \Omega_y u_{40} - \Omega u_{41}),\\
\Omega(83) &= (D_x)^{-1}(\Omega_y - \Omega u_{40}).
\end{aligned}$$

From these relations the recursion operator for symmetries $R^S(1)$ is defined.

It is interesting to present this nonlocal form-valued symmetry in the classical operator notation:

$$R^S(1) = -(D_x)^{-1}(\Omega(91) + \Omega_y u_{10} - \Omega u_{10}u_{40}) + u_{10}(D_x)^{-1}(\Omega_y - \Omega u_{40}),$$

or

$$\begin{aligned}
R^S(1) = &-(D_x)^{-1}((D_x)^{-1}(3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2))\\
&+ \Omega_y u_{10} - \Omega u_{10}u_{40}) + u_{10}(D_x)^{-1}(\Omega_y - \Omega u_{40}),
\end{aligned}$$

or

$$\begin{aligned}
R^S(1) = &-(D_x)^{-1}((D_x)^{-1}(3(D_x)^{-1}(\Omega(76) + \Omega_y u_{30} + 3\Omega u_{31})\\
&+ \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2))\\
&+ \Omega_y u_{10} - \Omega u_{10}u_{40}) + u_{10}(D_x)^{-1}(\Omega_y - \Omega u_{40}),
\end{aligned}$$

or

$$\begin{aligned}
R^S(1) = &-(D_x)^{-1}((D_x)^{-1}(3(D_x)^{-1}((D_x)^{-1}(\Omega_{yy} - \Omega_y u_{40} - \Omega u_{41})\\
&+ \Omega_y u_{30} + 3\Omega u_{31}) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2))\\
&+ \Omega_y u_{10} - \Omega u_{10}u_{40}) + u_{10}(D_x)^{-1}(\Omega_y - \Omega u_{40}).
\end{aligned}$$

Finally, we arrive at

$$\begin{aligned}
R^S(1) = &-(D_x)^{-1}((D_x)^{-1}(3(D_x)^{-1}((D_x)^{-1}(\Omega_{yy} - u_{40}\Omega_y - u_{41}\Omega)\\
&+ u_{30}\Omega_y + 3u_{31}\Omega) + u_{20}\Omega_y + (-u_{20}u_{40} - 8u_{21} - u_{30}^2)\Omega)\\
&+ u_{10}\Omega_y - u_{10}u_{40}\Omega) + u_{10}(D_x)^{-1}(\Omega_y - u_{40}\Omega),
\end{aligned}$$

whereas

$$\Omega_{yy} = (D_y)^2, \quad \Omega_y = (D_y), \quad \Omega = I = \text{identity}.$$

By construction these formal operators are indeed well defined.

2.7. **Generating functions in ℓ-covering.** Here we describe *form-valued* generating functions, which represent the classical symplectic operators, sending symmetries into generating functions of the equation. These form-valued generating functions, GF = SMP, have to satisfy the condition

$$\ell_{\mathcal{E}}^*(\mathrm{GF}) = 0,$$

which results in the condition

$$-(D_y)^3(\mathrm{GF}) + 2(D_x)^2(D_y)(u_{21}\,\mathrm{GF}) - (D_x)^3(u_{12}\,\mathrm{GF}) - (D_x)(D_y)^2(u_{30}\,\mathrm{GF}) = 0.$$

We obtained the following solutions of this condition

$$GF(1)^u = \Omega_x,$$

$$GF(2)^u = -\Omega(91) + \Omega(83)u_{20},$$

in which

$$\Omega(91) = (D_x)^{-1}(3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)),$$
$$\Omega(84) = (D_x)^{-1}(\Omega(76) + \Omega_y u_{30} + 3\Omega u_{31}),$$
$$\Omega(76) = (D_x)^{-1}(\Omega_{yy} - \Omega_y u_{40} - \Omega u_{41}),$$
$$\Omega(83) = (D_x)^{-1}(\Omega_y - \Omega u_{40}).$$

It is interesting to present these nonlocal form-valued generating guncutions in the classical operator notation: note that the *first symplectic structure* is *local* and is given as

$$\mathrm{SMP}(1) = D_x,$$

while the second symplectic structure is formally denoted by

$$\mathrm{SMP}(2) = -(D_x)^{-1}(3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)) + u_{20}(D_x)^{-1}(\Omega_y - \Omega u_{40}),$$

or

$$\mathrm{SMP}(2) = -(D_x)^{-1}(3(D_x)^{-1}(\Omega(76) + \Omega_y u_{30} + 3\Omega u_{31}) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)) + u_{20}(D_x)^{-1}(\Omega_y - \Omega u_{40}),$$

or

$$\mathrm{SMP}(2) = -(D_x)^{-1}(3(D_x)^{-1}((D_x)^{-1}(\Omega_{yy} - \Omega_y u_{40} - \Omega u_{41}) + \Omega_y u_{30} + 3\Omega u_{31}) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)) + u_{20}(D_x)^{-1}(\Omega_y - \Omega u_{40}).$$

Using

$$\Omega_{yy} = (D_y)^2, \quad \Omega_y = (D_y), \quad \Omega = I = \text{identity},$$

we obtain

$$\mathrm{SMP}(2) = -(D_x)^{-1}(3(D_x)^{-1}((D_x)^{-1}(\Omega_{yy} - u_{40}\Omega_y - u_{41}\Omega) + u_{30}\Omega_y + 3u_{31}\Omega) + u_{20}\Omega_y + (-u_{20}u_{40} - 8u_{21} - u_{30}^2)\Omega) + u_{20}(D_x)^{-1}(\Omega_y - u_{40}\Omega),$$

or, equivalently

$$\mathrm{SMP}(2) = -(D_x)^{-1}(3(D_x)^{-1}((D_x)^{-1}((D_y)^2 - u_{40}(D_y) - u_{41}I) + u_{30}(D_y) + 3u_{31}I) + u_{20}(D_y) + (-u_{20}u_{40} - 8u_{21} - u_{30}^2)I) + u_{20}(D_x)^{-1}((D_y) - u_{40}I).$$

Due to the construction of the form-valued nonlocalities all operators are well defined.

2.8. **ℓ^*-covering.** In order to construct Hamiltonian operators and recursion operators for generating functions we have to construct the ℓ^*-covering of the Monge–Ampère equation, given by u and odd variables Π, Π_x, Π_y, Π_{xx}, Π_{xy}, $\Pi_{yy}, \ldots$ satisfying the condition

$$\ell_{\mathcal{E}}^*(\Pi) = 0,$$

which amounts to

$$-(D_y)^3(\Pi) + 2(D_x)^2(D_y)(u_{21}\Pi) - (D_x)^3(u_{12}\Pi) - (D_x)(D_y)^2(u_{30}\Pi) = 0.$$

We have to construct nonlocalities in this ℓ^*-covering first, the results of which are presented in the next subsection.

2.9. **Conservation laws and nonlocalities in ℓ^*-covering.** Here we construct *vector-valued* nonlocalities, which are given by $\Pi(76)$, $\Pi(77),\dots$ They result from nontrivial vector-valued conservation laws in the ℓ^*-covering of the Monge–Ampère equation.

The vector-valued nonlocalities can be constructed from conservation laws

$$D_x(F^y) - D_y(F^x) = 0,$$

where we choose the presentation

$$\begin{aligned}
F^x &= S\Pi_{yy} + a_y\Pi_y + a_0\Pi,\\
F^y &= b_{yy}\Pi_{yy} + b_{xy}\Pi_{xy} + b_{xx}\Pi_{xx} + b_y\Pi_y + b_x\Pi_x + b_0\Pi,
\end{aligned}$$

where

$$\begin{aligned}
b_{yy} &= -u_{30}S,\\
b_{xy} &= 2u_{21}S,\\
b_{xx} &= -u_{12}S,\\
b_y &= -D_x(b_{xy}) + 2u_{31}S,\\
b_x &= -D_x(b_{xx}) - u_{22}S,\\
b_0 &= -D_x(b_x),\\
a_y &= -D_y(S) + D_x(b_{yy}) + u_{40}S,\\
a_0 &= -D_y(a_y) + D_x(b_y),
\end{aligned}$$

while S is a solution of

$$\ell_{\mathcal{E}}(S) = 0,$$

which amounts to

$$(D_y)^3(S) - 2u_{xxy}(D_x)^2(D_y)(S) + u_{xyy}(D_x)^3(S) + u_{xxx}(D_x)(D_y)^2(S) = 0,$$

i.e., S is a symmetry of the Monge–Ampère equation.

This presentation holds for nonlocalities of depth zero; for higher nonlocalities the theoretical presentation is more complicated. These vector-valued conservation laws can be computed in a straightforward way, leading to *vector-valued* nonlocalities $\Pi(76),\dots,\Pi(100)$, defined by their x and y derivatives. Those necessary for presentation of the results are:

$$\begin{aligned}
D_x\Pi(76) &= \Pi_{yy},\\
D_y\Pi(76) &= -\Pi_{yy}u_{30} + 2\Pi_{xy}u_{21} - \Pi_{xx}u_{12},\\
D_x\Pi(77) &= -\Pi_{yy}u_{10} + \Pi_y(u_{11} + u_{20}u_{30}) + \Pi(-u_{12} + u_{20}u_{31} + u_{21}u_{30}),\\
D_y\Pi(77) &= \Pi_{yy}u_{10}u_{30} - 2\Pi_{xy}u_{10}u_{21} + \Pi_{xx}u_{10}u_{12} + 2\Pi_y u_{20}u_{21}\\
&\quad - \Pi_x u_{12}u_{20} + \Pi(u_{12}u_{30} + u_{20}u_{22}),\\
D_x\Pi(78) &= -\Pi_{yy}u_{01} + \Pi_y(u_{02} + u_{11}u_{30}) + \Pi(u_{11}u_{31} + u_{21}^2),\\
D_y\Pi(78) &= \Pi_{yy}u_{01}u_{30} - 2\Pi_{xy}u_{01}u_{21} + \Pi_{xx}u_{01}u_{12} + 2\Pi_y u_{11}u_{21}\\
&\quad - \Pi_x u_{11}u_{12} + \Pi(u_{11}u_{22} + u_{12}u_{21}),\\
D_x\Pi(83) &= \Pi(76) + \Pi_y u_{30} + \Pi u_{31},\\
D_y\Pi(83) &= 2\Pi_y u_{21} - \Pi_x u_{12} + \Pi u_{22},\\
D_x\Pi(84) &= \Pi(77) - 4\Pi_{yy}u + \Pi_y(4u_{01} + 3u_{10}u_{30})\\
&\quad + \Pi(-4u_{02} + 3u_{10}u_{31} - 3u_{11}u_{30} + 4u_{20}u_{21}),\\
D_y\Pi(84) &= 4\Pi_{yy}uu_{30} - 8\Pi_{xy}uu_{21} + 4\Pi_{xx}uu_{12} + 6\Pi_y u_{10}u_{21} - 3\Pi_x u_{10}u_{12}\\
&\quad + \Pi(3u_{10}u_{22} + 2u_{12}u_{20}),
\end{aligned}$$

$$
\begin{aligned}
D_x\Pi(88) &= \Pi_y,\\
D_y\Pi(88) &= \Pi(76),\\
D_x\Pi(89) &= \Pi_{yy}u + \Pi_y(-2u_{01} - u_{10}u_{30})\\
&\quad + \Pi(3u_{02} - u_{10}u_{31} + 2u_{11}u_{30} - 2u_{20}u_{21}),\\
D_y\Pi(89) &= \Pi(78) - \Pi_{yy}uu_{30} + 2\Pi_{xy}uu_{21} - \Pi_{xx}uu_{12} - 2\Pi_y u_{10}u_{21}\\
&\quad + \Pi_x u_{10}u_{12} - \Pi(u_{10}u_{22} + u_{12}u_{20}),\\
D_x\Pi(91) &= \Pi,\\
D_y\Pi(91) &= \Pi(88),\\
D_x\Pi(92) &= \Pi(88) + \Pi u_{30},\\
D_y\Pi(92) &= \Pi(83),\\
D_x\Pi(93) &= \Pi(83) - 2\Pi u_{21},\\
D_y\Pi(93) &= -\Pi u_{12},\\
D_x\Pi(97) &= 3\Pi(93) + \Pi(88)u_{20} + \Pi(2u_{11} + u_{20}u_{30}),\\
D_y\Pi(97) &= \Pi(88)u_{11} + \Pi(77) + \Pi(76)u_{10},\\
D_x\Pi(99) &= \Pi(97) + \Pi(88)u_{10} + \Pi(2u_{01} + u_{10}u_{30}),\\
D_y\Pi(99) &= 2\Pi(89) + 2\Pi(88)u_{01} + \Pi(84) + 2\Pi(76)u.
\end{aligned}
$$

This finishes the construction of the ℓ^*-covering.

2.10. **Symmetries in ℓ^*-Covering.** We are going to search for *Hamiltonian operators* as Π-valued solutions H of

$$\ell_{\mathcal{E}}(H) = 0.$$

This leads to the following results

$$
\begin{aligned}
H(0) &= -\Pi(91),\\
H(1) &= -\Pi(99) + \Pi(92)u_{10} + 2\Pi(91)u_{01},
\end{aligned}
$$

where $\Pi(99)$, $\Pi(92)$, $\Pi(91)$, $\Pi(97)$, $\Pi(93)$, $\Pi(88)$, $\Pi(83)$, $\Pi(76)$ are formally defined by

$$
\begin{aligned}
\Pi(99) &= (D_x)^{-1}(\Pi(97) + \Pi(88)u_{10} + \Pi(2u_{01} + u_{10}u_{30})),\\
\Pi(92) &= (D_x)^{-1}(\Pi(88) + \Pi u_{30}),\\
\Pi(91) &= (D_x)^{-1}(\Pi),\\
\Pi(97) &= (D_x)^{-1}(3\Pi(93) + \Pi(88)u_{20} + \Pi(2u_{11} + u_{20}u_{30})),\\
\Pi(93) &= (D_x)^{-1}(\Pi(83) - 2\Pi u_{21}),\\
\Pi(88) &= (D_x)^{-1}(\Pi_y),\\
\Pi(83) &= (D_x)^{-1}(\Pi(76) + \Pi_y u_{30} + \Pi u_{31}),\\
\Pi(76) &= (D_x)^{-1}(\Pi_{yy}),
\end{aligned}
$$

and from which the operator notation for $H(0)$ and $H(1)$ can be obtained in a similar way as recursion operator for symmetries and symplectic operator of previous sections. For the presentation of $H(2)$, the reader is referred to the Appendix.

2.11. **Generating functions in ℓ^*-covering.** Here we describe *vector-valued* generating functions, which represent the classical recursion operators for generating functions, R^G. These vector-valued generating functions have to satisfy the condition

$$\ell^*_{\mathcal{E}}(R^G) = 0,$$

which results in the condition

$$-(D_y)^3(R^G) + 2(D_x)^2(D_y)(u_{21}R^G) - (D_x)^3(u_{12}R^G) - (D_x)(D_y)^2(u_{30}R^G) = 0.$$

We obtained three vector-valued generating functions

$$\begin{aligned} R^G(0) &= \Pi, \\ R^G(1) &= \Pi(97) - \Pi(92)u_{20} - 2\Pi(91)u_{11}, \end{aligned}$$

where of course $R^G(0)$ is just the identity operator. For the presentation of $R^G(2)$, the reader is referred to the Appendix.

2.12. **Transformation of form-valued results.** We can choose another presentation of the nonlocal form-valued nonlocalities which is performed by partial integrations, thus removing deeper nonlocal behaviour, and thus transforming from $\Omega(76), \dots, \Omega(94)$ into $\Omega(101), \dots, \Omega(119)$. These form-valued nonlocalities can be constructed from conservation laws

$$D_x(F^y) - D_y(F^x) = 0,$$

where we choose the presentation

$$\begin{aligned} F^x &= G\Omega_{yy} + a_y\Omega_y + a_0\Omega, \\ F^y &= b_{yy}\Omega_{yy} + b_{xy}\Omega_{xy} + b_{xx}\Omega_{xx} + b_y\Omega_y + b_x\Omega_x + b_0\Omega, \end{aligned}$$

where

$$\begin{aligned} b_{yy} &= -u_{30}G \\ b_{xy} &= 2u_{21}G \\ b_{xx} &= -u_{12}G \\ b_y &= -D_x(b_{xy}) \\ b_x &= -D_x(b_{xx}) \\ b_0 &= -D_x(b_x) \\ a_y &= -D_y(G) + D_x(b_{yy}) \\ a_0 &= -D_y(a_y) + D_x(b_y) \end{aligned}$$

while G is a solution of

$$\ell^*_{\mathcal{E}}(G) = 0,$$

which amounts to

$$-(D_y)^3(G) + 2(D_x)^2(D_y)(u_{21}G) - (D_x)^3(u_{12}G) - (D_x)(D_y)^2(u_{30}G) = 0,$$

i.e., G is a generating function. The transformation is given by

$$\begin{aligned} \Omega(76) &= \Omega(101), \\ \Omega(77) &= \Omega(102), \\ \Omega(78) &= \Omega(103), \\ \Omega(83) &= \Omega(108) + \Omega(101)y, \\ \Omega(84) &= \Omega(109) + \Omega(101)x, \\ \Omega(85) &= \Omega(110) + \Omega(102)x, \\ \Omega(86) &= \Omega(111) + \Omega(103)y, \\ \Omega(91) &= (2\Omega(116) + 6\Omega(109)x + 2\Omega(102)y + 3\Omega(101)x^2)/2, \\ \Omega(94) &= (2\Omega(119) + 2\Omega(116)x + 4\Omega(111)y + 2\Omega(110)y + 3\Omega(109)x^2 \end{aligned}$$

$$+ 2\Omega(103)y^2 + 2\Omega(102)xy + \Omega(101)x^3)/2.$$

The x and y derivatives of these nonlocal form-valued variables fit perfectly in the canonical presentation and are obtained from functions G, given by

$$\begin{aligned}
G_{\Omega(101)} &= 1,\\
G_{\Omega(102)} &= u_{20},\\
G_{\Omega(103)} &= u_{11},\\
G_{\Omega(108)} &= -y,\\
G_{\Omega(109)} &= -x,\\
G_{\Omega(110)} &= -xu_{20} + 3u_{10},\\
G_{\Omega(111)} &= -yu_{11} - u_{10},\\
G_{\Omega(116)} &= (3x^2 - 2yu_{20})/2,\\
G_{\Omega(119)} &= (-x^3 + 2xyu_{20} + 2y^2u_{11} - 2yu_{10})/2.
\end{aligned}$$

Note that we only presented those vaiables necessary for transforming our results (full details are in Appendix).

Having performed this transformation , the recursion operator for symmetries $\overline{R}^S(1)$ and the second symplectic operator $\overline{\mathrm{SMP}}(2)$ are presented by

$$\begin{aligned}
\overline{R}^S(1) &= -2\Omega(119) - 2\Omega(116)x - 4\Omega(111)y - 2\Omega(110)y - 3\Omega(109)x^2\\
&\quad + 2\Omega(108)u_{10} - 2\Omega(103)y^2 - 2\Omega(102)xy + \Omega(101)(2u_{10}y - x^3),\\
\overline{SMP}(2) &= -2\Omega(116) - 6\Omega(109)x + 2\Omega(108)u_{20} - 2\Omega(102)y\\
&\quad + \Omega(101)(2u_{20}y - 3x^2).
\end{aligned}$$

Due to the definitions of $\Omega(i)$, $i = 101, \ldots, 119$ the recursion operator for symmetries and the second symplectic operator can be nicely presented in terms of pseudo-differential operators, i.e., due to

$$\begin{aligned}
D_x\Omega(101) &= \Omega_{yy} - \Omega_y u_{40} - \Omega u_{41},\\
D_x\Omega(102) &= \Omega_{yy}u_{20} - \Omega_y(u_{20}u_{40} + u_{21} + u_{30}^2)\\
&\quad + \Omega(-u_{20}u_{41} - u_{21}u_{40} + u_{22} - 2u_{30}u_{31}),\\
D_x\Omega(103) &= \Omega_{yy}u_{11} - \Omega_y(u_{11}u_{40} + u_{12} + u_{21}u_{30}) + \Omega(-u_{11}u_{41} - 3u_{21}u_{31}),\\
D_x\Omega(108) &= -\Omega_{yy}y + \Omega_y(u_{40}y + 1) + \Omega(-u_{40} + u_{41}y),\\
D_x\Omega(109) &= -\Omega_{yy}x + \Omega_y(u_{30} + u_{40}x) + \Omega(3u_{31} + u_{41}x),\\
D_x\Omega(110) &= \Omega_{yy}(3u_{10} - u_{20}x)\\
&\quad + \Omega_y(-3u_{10}u_{40} - 3u_{11} - 2u_{20}u_{30} + u_{20}u_{40}x + u_{21}x + u_{30}^2x)\\
&\quad + \Omega(-3u_{10}u_{41} + 3u_{11}u_{40} + 3u_{12}\\
&\quad - 6u_{20}u_{31} + u_{20}u_{41}x + u_{21}u_{40}x - u_{22}x + 2u_{30}u_{31}x),\\
D_x\Omega(111) &= -\Omega_{yy}(u_{10} + u_{11}y)\\
&\quad + \Omega_y(u_{10}u_{40} + u_{11}u_{40}y + 2u_{11} + u_{12}y + u_{20}u_{30} + u_{21}u_{30}y)\\
&\quad + \Omega(u_{10}u_{41} - 2u_{11}u_{40} + u_{11}u_{41}y - 3u_{12} + 3u_{20}u_{31} + 3u_{21}u_{31}y),\\
D_x\Omega(116) &= (\Omega_{yy}(-2u_{20}y + 3x^2)\\
&\quad + \Omega_y(2u_{20}u_{40}y + 2u_{20} + 2u_{21}y + 2u_{30}^2y - 6u_{30}x - 3u_{40}x^2)\\
&\quad + \Omega(-2u_{20}u_{40} + 2u_{20}u_{41}y + 2u_{21}u_{40}y - 16u_{21} - 2u_{22}y - 2u_{30}^2\\
&\quad + 4u_{30}u_{31}y - 18u_{31}x - 3u_{41}x^2))/2,\\
D_x\Omega(119) &= (\Omega_{yy}(-2u_{10}y + 2u_{11}y^2 + 2u_{20}xy - x^3)
\end{aligned}$$

$$+ \Omega_y(2u_{10}u_{40}y + 2u_{10} - 2u_{11}u_{40}y^2 - 2u_{11}y - 2u_{12}y^2 - 2u_{20}u_{40}xy - 2u_{20}x - 2u_{21}u_{30}y^2 - 2u_{21}xy - 2u_{30}^2xy + 3u_{30}x^2 + u_{40}x^3) + \Omega(-2u_{10}u_{40} + 2u_{10}u_{41}y + 2u_{11}u_{40}y - 2u_{11}u_{41}y^2 + 6u_{12}y + 2u_{20}u_{40}x - 2u_{20}u_{41}xy - 6u_{21}u_{31}y^2 - 2u_{21}u_{40}xy + 16u_{21}x + 2u_{22}xy + 2u_{30}^2x - 4u_{30}u_{31}xy + 9u_{31}x^2 + u_{41}x^3))/2.$$

The second symplectic operator $\overline{\mathrm{SMP}}(2)$ is *completely formally* represented as

$$\overline{\mathrm{SMP}}(2) = -2(D_x)^{-1}((\Omega_{yy}(-2u_{20}y + 3x^2) + \Omega_y(2u_{20}u_{40}y + 2u_{20} + 2u_{21}y + 2u_{30}^2y - 6u_{30}x - 3u_{40}x^2) + \Omega(-2u_{20}u_{40} + 2u_{20}u_{41}y + 2u_{21}u_{40}y - 16u_{21} - 2u_{22}y - 2u_{30}^2 + 4u_{30}u_{31}y - 18u_{31}x - 3u_{41}x^2))/2) - 6x(D_x)^{-1}(-\Omega_{yy}x + \Omega_y(u_{30} + u_{40}x) + \Omega(3u_{31} + u_{41}x)) + 2u_{20}(D_x)^{-1}(-\Omega_{yy}y + \Omega_y(u_{40}y + 1) + \Omega(-u_{40} + u_{41}y)) - 2y(D_x)^{-1}(\Omega_{yy}u_{20} - \Omega_y(u_{20}u_{40} + u_{21} + u_{30}^2) + \Omega(-u_{20}u_{41} - u_{21}u_{40} + u_{22} - 2u_{30}u_{31})) + (2u_{20}y - 3x^2)(D_x)^{-1}(\Omega_{yy} - \Omega_y u_{40} - \Omega u_{41})$$

and similarly for the recursion operator for symmetries, $R^S(1)$.

2.13. **Transformation of vector-valued results.** We can choose another presentation of the nonlocal vector-valued nonlocalities which is performed by partial integrations, thus removing deeper nonlocal behaviour, and transforming from $\Omega(76), \ldots, \Omega(100)$ into $\Omega(101), \ldots \Omega(125)$. The vector-valued nonlocalities can be constructed from conservation laws

$$D_x(F^y) - D_y(F^x) = 0,$$

where we choose the presentation

$$F^x = S\Pi_{yy} + a_y\Pi_y + a_0\Pi,$$
$$F^y = b_{yy}\Pi_{yy} + b_{xy}\Pi_{xy} + b_{xx}\Pi_{xx} + b_y\Pi_y + b_x\Pi_x + b_0\Pi,$$

where

$$b_{yy} = -u_{30}S,$$
$$b_{xy} = 2u_{21}S,$$
$$b_{xx} = -u_{12}S,$$
$$b_y = -D_x(b_{xy}) + 2u_{31}S,$$
$$b_x = -D_x(b_{xx}) - u_{22}S,$$
$$b_0 = -D_x(b_x),$$
$$a_y = -D_y(S) + D_x(b_{yy}) + u_{40}S,$$
$$a_0 = -D_y(a_y) + D_x(b_y),$$

while S is a solution of

$$\ell_{\mathcal{E}}(S) = 0,$$

which amounts to

$$(D_y)^3(S) - 2u_{xxy}(D_x)^2(D_y)(S) + u_{xyy}(D_x)^3(S) + u_{xxx}(D_x)(D_y)^2(S) = 0,$$

i.e., S is a symmetry. We made the following transformation:

$$\Pi(76) = \Pi(101),$$

$$\begin{aligned}
\Pi(83) &= \Pi(108) + \Pi(101)x,\\
\Pi(88) &= \Pi(113) + \Pi(101)y,\\
\Pi(91) &= (2\Pi(116) + 2\Pi(113)y + \Pi(101)y^2)/2,\\
\Pi(92) &= \Pi(117) + \Pi(113)x + \Pi(108)y + \Pi(101)xy,\\
\Pi(97) &= (2\Pi(122) + 6\Pi(118)x + 2\Pi(113)u_{10} + 3\Pi(108)x^2\\
&\quad + 2\Pi(102)y + \Pi(101)x^3 + 2\Pi(101)yu_{10})/2,\\
\Pi(99) &= (8\Pi(124) + 8\Pi(122)x + 12\Pi(118)x^2\\
&\quad + 16\Pi(114)y + 16\Pi(113)u + 8\Pi(109)y + 4\Pi(108)x^3\\
&\quad + 8\Pi(103)y^2 + 8\Pi(102)xy + \Pi(101)x^4 + 16\Pi(101)yu)/8.
\end{aligned}$$

The x and y derivatives of these nonlocal vector-valued variables fit perfectly in the canonical presentation and are obtained from functions S, given by

$$\begin{aligned}
S_{\Pi_{101}} &= 1,\\
S_{\Pi_{102}} &= -u_{10},\\
S_{\Pi_{103}} &= -u_{01},\\
S_{\Pi_{108}} &= -x,\\
S_{\Pi_{109}} &= xu_{10} - 4u,\\
S_{\Pi_{113}} &= -y,\\
S_{\Pi_{114}} &= yu_{01} + u,\\
S_{\Pi_{116}} &= y^2/2,\\
S_{\Pi_{117}} &= xy,\\
S_{\Pi_{118}} &= x^2/2,\\
S_{\Pi_{122}} &= (-x^3 + 2yu_{10})/2,\\
S_{\Pi_{124}} &= (x^4 - 8xyu_{10} - 8y^2u_{01} + 16yu)/8.
\end{aligned}$$

We should note that some of these (degree 4 in x) have not been presented before: $S_{\Pi(124)}$, $S_{\Pi(125)}$. Note that we only presented those vaiables necessary for transforming our results (full details are in Appendix). Having performed this transformation then Hamiltonian operators are represented by

$$\begin{aligned}
\overline{H}(0) &= 2\Pi(116) + 2\Pi(113)y + \Pi(101)y^2,\\
\overline{H}(1) &= -8\Pi(124) - 8\Pi(122)x - 12\Pi(118)x^2 + 8\Pi(117)u_{10} + 16\Pi(116)u_{01}\\
&\quad - 16\Pi(114)y + \Pi(113)(8xu_{10} + 16yu_{01} - 16u) - 8\Pi(109)y\\
&\quad + \Pi(108)(-4x^3 + 8yu_{10}) - 8\Pi(103)y^2 - 8\Pi(102)xy\\
&\quad + \Pi(101)(-x^4 + 8xyu_{10} + 8y^2u_{01} - 16yu),
\end{aligned}$$

while and recursion operators for generating functions are represented by

$$\begin{aligned}
\overline{R}^G(0) &= \Pi,\\
\overline{R}^G(1) &= 2\Pi(122) + 6\Pi(118)x - 2\Pi(117)u_{20} - 4\Pi(116)u_{11}\\
&\quad + 2\Pi(113)(u_{10} - 2u_{11}y - u_{20}x) + \Pi(108)(-2u_{20}y + 3x^2)\\
&\quad + 2\Pi(102)y + \Pi(101)(2u_{10}y - 2u_{11}y^2 - 2u_{20}xy + x^3).
\end{aligned}$$

In a way similar as before it is possible to rewrite these operator in classical terms of pseudo-differential operators, but we shall not pursue this here.There are, due to deeper nonlocalities, in the operator notation of the vector-valued results higher pseudo-differential operators.

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3. Appendix

3.1. Conservation laws and nonlocalities.

3.1.1. *Zero-order.*

$$
\begin{aligned}
(Q_{0,7})_x &= -u_{01}u_{40} + u_{02}\\
(Q_{0,7})_y &= -u_{01}u_{31} - u_{02}u_{30} + u_{11}u_{21},\\
(Q_{0,9})_x &= 2u_{02}u_{20} + u_{11}^2 - u_{20}^2u_{21}\\
(Q_{0,9})_y &= 2u_{02}u_{11} - u_{12}u_{20}^2,\\
(Q_{0,12})_x &= u_{11}(u_{02} - u_{20}u_{21})\\
(Q_{0,12})_y &= (u_{02}^2 - 2u_{11}u_{12}u_{20})/2.
\end{aligned}
$$

3.1.2. *First order.*

$$
\begin{aligned}
(Q_{1,3})_x &= 2u_{01} - u_{20}^2\\
(Q_{1,3})_y &= 2(Q_{0,7} + u_{01}u_{30} - u_{11}u_{20})\\
(Q_{1,6})_x &= Q_{0,7} + u_{01}u_{30}\\
(Q_{1,6})_y &= u_{11}^2/2\\
(Q_{1,8})_x &= (Q_{0,9} - 2u_{01}u_{10}u_{40} - 4u_{01}u_{20}u_{30} + 2u_{02}u_{10})\\
(Q_{1,8})_y &= (4Q_{0,12} - 2u_{01}u_{10}u_{31} - 2u_{01}u_{11}u_{30} - 2u_{01}u_{20}u_{21} - 2u_{02}u_{10}u_{30}\\
&\quad - u_{02}u_{20}^2 + 2u_{10}u_{11}u_{21} + 2u_{11}^2u_{20}),\\
(Q_{1,11})_x &= 4Q_{0,12} + u_{01}^2u_{40} - 2u_{01}u_{02} + u_{01}u_{20}^2u_{40} + 2u_{01}u_{20}u_{30}^2\\
&\quad - u_{02}u_{20}^2 + u_{11}^2u_{20}\\
(Q_{1,11})_y &= u_{01}^2u_{31} + 2u_{01}u_{02}u_{30} - 2u_{01}u_{11}u_{21} + u_{01}u_{20}^2u_{31} + 2u_{01}u_{20}u_{21}u_{30}\\
&\quad - 2u_{02}u_{11}u_{20} + u_{02}u_{20}^2u_{30} + u_{11}^3 - u_{11}u_{20}^2u_{21},\\
(Q_{1,13})_x &= 216Q_{0,7}^2 + 432Q_{0,7}u_{01}u_{30} - 216u_{01}^2u_{20}u_{40} - 93u_{01}^2u_{21} - 12u_{01}u_{02}u_{20}\\
&\quad + 10u_{01}u_{10}^2u_{41} + 20u_{01}u_{10}u_{11}u_{40} - 156u_{01}u_{10}u_{12} - 114u_{01}u_{11}^2\\
&\quad + 114u_{01}u_{20}^2u_{21} + 12u_{02}u_{10}^2u_{40} + 132u_{02}u_{10}u_{11}\\
&\quad + 12u_{02}u_{20}^3 - 2u_{10}^3u_{32} + 10u_{10}^2u_{11}u_{31} + 18u_{10}^2u_{12}u_{30} - 348u_{10}u_{11}u_{20}u_{21}\\
&\quad + 6u_{11}^2u_{20}^2 - 6u_{20}^4u_{21}\\
(Q_{1,13})_y &= 216Q_{0,7}u_{11}^2 - 93u_{01}^2u_{12} - 216u_{01}^2u_{20}u_{31} - 216u_{01}^2u_{21}u_{30} - 198u_{01}u_{02}u_{11}\\
&\quad - 432u_{01}u_{02}u_{20}u_{30} + 10u_{01}u_{10}^2u_{32} + 40u_{01}u_{10}u_{11}u_{31} + 176u_{01}u_{10}u_{12}u_{30}\\
&\quad - 20u_{01}u_{10}u_{20}u_{22} - 176u_{01}u_{10}u_{21}^2 + 236u_{01}u_{11}^2u_{30} + 392u_{01}u_{11}u_{20}u_{21}\\
&\quad + 134u_{01}u_{12}u_{20}^2 - 12u_{02}^2u_{10} + 22u_{02}u_{10}^2u_{31} + 44u_{02}u_{10}u_{11}u_{30}\\
&\quad - 44u_{02}u_{10}u_{20}u_{21} + 194u_{02}u_{11}u_{20}^2 + 2u_{10}^3u_{12}u_{50} - 4u_{10}^3u_{21}u_{41}\\
&\quad + 4u_{10}^3u_{22}u_{40}\\
&\quad + 2u_{10}^3u_{30}u_{32} - 4u_{10}^3u_{31}^2 - 6u_{10}^2u_{11}u_{22} - 6u_{10}^2u_{12}u_{20}u_{40} + 6u_{10}^2u_{12}u_{21}\\
&\quad - 12u_{10}^2u_{12}u_{30}^2 + 12u_{10}^2u_{20}u_{21}u_{31} - 6u_{10}^2u_{20}u_{22}u_{30} + 12u_{10}^2u_{21}^2u_{30}\\
&\quad - 20u_{10}u_{11}^2u_{21} - 316u_{10}u_{11}u_{12}u_{20} + 12u_{10}u_{12}u_{20}^2u_{30} - 12u_{10}u_{20}^2u_{21}^2\\
&\quad - 236u_{11}^3u_{20} - 6u_{12}u_{20}^4,\\
(Q_{1,16})_x &= -18Q_{0,9}u_{02} - 18Q_{0,9}u_{11}u_{30} + 20u_{01}u_{02}u_{10}u_{40} + 72u_{01}u_{02}u_{11}\\
&\quad - 5u_{01}u_{10}^2u_{32} + 30u_{01}u_{10}u_{12}u_{30} - 72u_{01}u_{11}u_{20}u_{21} + 10u_{02}u_{10}^2u_{31}\\
&\quad - 48u_{02}u_{11}u_{20}^2 + 15u_{10}^2u_{12}u_{21} - 36u_{11}^3u_{20} + 12u_{11}u_{20}^3u_{21},\\
(Q_{1,16})_y &= (-36Q_{0,9}u_{11}u_{21} + 72u_{01}u_{02}^2 + 40u_{01}u_{02}u_{10}u_{31} + 40u_{01}u_{02}u_{11}u_{30}\\
&\quad - 40u_{01}u_{02}u_{20}u_{21} + 10u_{01}u_{10}^2u_{12}u_{50} - 20u_{01}u_{10}^2u_{21}u_{41} + 20u_{01}u_{10}^2u_{22}u_{40}\\
&\quad + 10u_{01}u_{10}^2u_{30}u_{32} - 20u_{01}u_{10}^2u_{31}^2 - 20u_{01}u_{10}u_{11}u_{22} - 20u_{01}u_{10}u_{12}u_{20}u_{40}\\
&\quad + 20u_{01}u_{10}u_{12}u_{21} - 40u_{01}u_{10}u_{12}u_{30}^2 + 40u_{01}u_{10}u_{20}u_{21}u_{31}\\
&\quad - 20u_{01}u_{10}u_{20}u_{22}u_{30}\\
&\quad + 40u_{01}u_{10}u_{21}^2u_{30} - 124u_{01}u_{11}u_{12}u_{20} + 20u_{01}u_{12}u_{20}^2u_{30}\\
&\quad - 20u_{01}u_{20}^2u_{21}^2\\
&\quad + 40u_{02}^2u_{10}u_{30} - 20u_{02}^2u_{20}^2 + 10u_{02}u_{10}^2u_{22} - 20u_{02}u_{10}u_{12}u_{20}\\
&\quad - 112u_{02}u_{11}^2u_{20}\\
&\quad - 10u_{10}^2u_{11}u_{12}u_{40} + 20u_{10}^2u_{11}u_{21}u_{31} - 10u_{10}^2u_{11}u_{22}u_{30} + 10u_{10}^2u_{12}^2\\
&\quad - 20u_{10}^2u_{12}u_{21}u_{30} + 20u_{10}^2u_{21}^3 + 20u_{10}u_{11}^2u_{12} + 40u_{10}u_{11}u_{12}u_{20}u_{30}
\end{aligned}
$$

$$- 40u_{10}u_{11}u_{20}u_{21}^2 - 9u_{11}^4 + 24u_{11}u_{12}u_{20}^3)/2.$$

3.1.3. *Second order.*

$$\begin{aligned}
(Q_{2,5})_x &= -18Q_{1,6} - 2u_{01}u_{20} + 4u_{10}u_{11} + u_{20}^3\\
(Q_{2,5})_y &= -3Q_{0,9} - 2u_{01}u_{11} + 4u_{02}u_{10}\\
(Q_{2,7})_x &= (-80Q_{0,7}u_{10} - 20Q_{1,8} - 20u_{01}^2 - 120u_{01}u_{10}u_{30} + 2u_{10}u_{20}^2u_{30} - u_{20}^4)/2\\
(Q_{2,7})_y &= -40Q_{0,7}u_{01} - 10Q_{1,11} - 30u_{01}^2u_{30} + 20u_{01}u_{11}u_{20} + 10u_{01}u_{20}^2u_{30}\\
&\quad - 30u_{10}u_{11}^2 + u_{10}u_{20}^2u_{21} - 3u_{11}u_{20}^3\\
(Q_{2,10})_x &= 2Q_{0,7}u_{01} - Q_{0,7}u_{20}^2 - 2Q_{1,11} - 3Q_{1,3}u_{01}u_{40} + 3Q_{1,3}u_{02} + 4u_{01}^2u_{30}\\
&\quad - 18u_{01}u_{10}u_{20}u_{40} - 6u_{01}u_{10}u_{21} - 18u_{01}u_{10}u_{30}^2 + 6u_{01}u_{11}u_{20}\\
&\quad - 44u_{01}u_{20}^2u_{30} + 10u_{02}u_{10}u_{20} + 3u_{10}^2u_{12}\\
&\quad - 4u_{10}u_{11}^2 + 7u_{10}u_{20}^2u_{21} - 3u_{11}u_{20}^3\\
(Q_{2,10})_y &= 4Q_{0,7}^2 + 2Q_{0,7}u_{01}u_{30} - 2Q_{0,7}u_{11}u_{20} - 3Q_{1,3}u_{01}u_{31} - 3Q_{1,3}u_{02}u_{30}\\
&\quad + 3Q_{1,3}u_{11}u_{21} + 6u_{01}^2u_{21} - 2u_{01}^2u_{30}^2 + 12u_{01}u_{02}u_{20} - 6u_{01}u_{10}u_{12}\\
&\quad - 18u_{01}u_{10}u_{20}u_{31} - 18u_{01}u_{10}u_{21}u_{30} - 6u_{01}u_{11}^2 - 14u_{01}u_{11}u_{20}u_{30}\\
&\quad - 30u_{01}u_{20}^2u_{21} + 4u_{02}u_{10}u_{11} - 18u_{02}u_{10}u_{20}u_{30} - 10u_{02}u_{20}^3 - 3u_{10}^2u_{12}u_{30}\\
&\quad + 3u_{10}^2u_{21}^2 + 18u_{10}u_{11}u_{20}u_{21} + 7u_{10}u_{12}u_{20}^2 + 7u_{11}^2u_{20}^2\\
(Q_{2,12})_x &= (-576Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} - 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2\\
&\quad + Q_{1,13} + 300u_{01}u_{02}u_{10} + 504u_{01}u_{10}^2u_{31} + 690u_{01}u_{10}u_{11}u_{30}\\
&\quad + 1614u_{01}u_{10}u_{20}u_{21} - 54u_{02}u_{10}^2u_{30} + 2u_{10}^4u_{51} - 12u_{10}^3u_{11}u_{50}\\
&\quad - 40u_{10}^3u_{21}u_{40} + 18u_{10}^3u_{22} - 36u_{10}^3u_{30}u_{31} + 36u_{10}^2u_{11}u_{30}^2)\\
(Q_{2,12})_y &= (-576Q_{0,12}u_{01} + 72Q_{0,7}Q_{0,9} + 144Q_{0,7}u_{01}u_{11} + 72Q_{0,9}u_{01}u_{30}\\
&\quad + 72Q_{0,9}u_{11}u_{20} + 8Q_{1,16} - 93u_{01}^2u_{02} + 144u_{01}^2u_{11}u_{30} - 360u_{01}^2u_{20}u_{21}\\
&\quad - 160u_{01}u_{02}u_{10}u_{30} - 280u_{01}u_{02}u_{20}^2 + 554u_{01}u_{10}^2u_{22} + 1738u_{01}u_{10}u_{11}u_{21}\\
&\quad + 486u_{01}u_{10}u_{12}u_{20} + 566u_{01}u_{11}^2u_{20} + 24u_{01}u_{20}^3u_{21} + 392u_{02}u_{10}^2u_{21}\\
&\quad + 786u_{02}u_{10}u_{11}u_{20} + 6u_{02}u_{20}^4 + 2u_{10}^4u_{42} - 4u_{10}^3u_{11}u_{41} - 28u_{10}^3u_{12}u_{40}\\
&\quad - 8u_{10}^3u_{20}u_{32} - 4u_{10}^3u_{21}u_{31} - 44u_{10}^3u_{22}u_{30} - 36u_{10}^2u_{11}^2u_{40} - 506u_{10}^2u_{11}u_{12}\\
&\quad + 12u_{10}^2u_{11}u_{20}u_{31} - 48u_{10}^2u_{11}u_{21}u_{30} + 78u_{10}^2u_{12}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{22}\\
&\quad + 6u_{10}^2u_{20}u_{21}^2 - 586u_{10}u_{11}^3 + 72u_{10}u_{11}^2u_{20}u_{30} - 24u_{10}u_{11}u_{20}^2u_{21}\\
&\quad - 48u_{10}u_{12}u_{20}^3 - 24u_{11}^2u_{20}^3)\\
(Q_{2,15})_x &= (-28800Q_{0,12}u_{01} + 14400Q_{0,12}u_{20}^2 - 7200Q_{0,7}Q_{0,9} - 14400Q_{0,7}u_{02}u_{10}\\
&\quad - 3600Q_{0,7}u_{10}^2u_{31} - 7200Q_{0,7}u_{10}u_{11}u_{30} - 7200Q_{0,9}u_{01}u_{30} + 800Q_{1,16}\\
&\quad - 7200u_{01}^2u_{02} + 7200u_{01}^2u_{20}u_{21} - 31200u_{01}u_{02}u_{10}u_{30} + 400u_{01}u_{10}^3u_{51}\\
&\quad - 3600u_{01}u_{10}^2u_{11}u_{50} + 1200u_{01}u_{10}^2u_{20}u_{41} - 7200u_{01}u_{10}^2u_{21}u_{40}\\
&\quad + 19200u_{01}u_{10}^2u_{22} - 7200u_{01}u_{10}^2u_{30}u_{31} + 21600u_{01}u_{10}u_{12}u_{20}\\
&\quad + 21600u_{01}u_{11}^2u_{20} + 300u_{02}u_{10}^3u_{50} - 22800u_{02}u_{10}^2u_{21} - 900u_{02}u_{10}^2u_{30}^2\\
&\quad - u_{10}^5u_{71} + 20u_{10}^4u_{21}u_{60} + 50u_{10}^4u_{30}u_{51} + 75u_{10}^4u_{31}u_{50} + 100u_{10}^4u_{40}u_{41}\\
&\quad - 25u_{10}^4u_{42} + 800u_{10}^3u_{12}u_{40} - 400u_{10}^3u_{21}u_{30}u_{40} - 1200u_{10}^3u_{21}u_{31}\\
&\quad + 600u_{10}^3u_{22}u_{30} - 300u_{10}^3u_{30}^2u_{31} - 9600u_{10}^2u_{11}^2u_{40}\\
&\quad - 18000u_{10}^2u_{11}u_{21}u_{30})/800\\
(Q_{2,15})_y &= (-28800Q_{0,12}Q_{0,7} - 28800Q_{0,12}u_{01}u_{30} + 28800Q_{0,12}u_{11}u_{20}
\end{aligned}$$

$$
\begin{aligned}
&- 3600Q_{0,7}u_{10}^2u_{22} - 14400Q_{0,7}u_{10}u_{11}u_{21} + 7200Q_{0,7}u_{10}u_{12}u_{20}\\
&- 7200Q_{0,7}u_{11}^2u_{20} - 10800Q_{0,9}u_{11}^2 + 7200u_{01}^2u_{12}u_{20} - 800u_{01}u_{02}u_{10}u_{21}\\
&- 800u_{01}u_{02}u_{11}u_{20} + 400u_{01}u_{10}^3u_{42} - 2400u_{01}u_{10}^2u_{11}u_{41}\\
&- 18800u_{01}u_{10}^2u_{12}u_{40} + 26800u_{01}u_{10}^2u_{21}u_{31} - 22400u_{01}u_{10}^2u_{22}u_{30}\\
&- 7200u_{01}u_{10}u_{11}^2u_{40} + 30400u_{01}u_{10}u_{11}u_{12} + 7200u_{01}u_{10}u_{11}u_{20}u_{31}\\
&- 14400u_{01}u_{10}u_{11}u_{21}u_{30} + 15200u_{01}u_{10}u_{12}u_{20}u_{30} - 800u_{01}u_{10}u_{20}u_{21}^2\\
&+ 7200u_{01}u_{11}^3 - 7200u_{01}u_{11}u_{20}^2u_{21} - 800u_{02}^2u_{10}u_{20} + 700u_{02}u_{10}^3u_{41}\\
&- 2700u_{02}u_{10}^2u_{11}u_{40} + 4000u_{02}u_{10}^2u_{12} - 900u_{02}u_{10}^2u_{20}u_{31}\\
&- 4500u_{02}u_{10}^2u_{21}u_{30} - 22400u_{02}u_{10}u_{11}^2 + 5400u_{02}u_{10}u_{11}u_{20}u_{30}\\
&+ 1800u_{02}u_{10}u_{20}^2u_{21} - 1800u_{02}u_{11}u_{20}^3 - u_{10}^5u_{62}\\
&- 5u_{10}^4u_{11}u_{61} + 25u_{10}^4u_{12}u_{60}\\
&+ 5u_{10}^4u_{20}u_{52} - 25u_{10}^4u_{21}u_{51} + 95u_{10}^4u_{22}u_{50} + 70u_{10}^4u_{30}u_{42} - 50u_{10}^4u_{31}u_{41}\\
&+ 130u_{10}^4u_{32}u_{40} + 20u_{10}^3u_{11}u_{20}u_{51} + 80u_{10}^3u_{11}u_{21}u_{50} + 180u_{10}^3u_{11}u_{30}u_{41}\\
&+ 220u_{10}^3u_{11}u_{31}u_{40} - 500u_{10}^3u_{11}u_{32} - 100u_{10}^3u_{12}u_{20}u_{50} - 800u_{10}^3u_{12}u_{30}u_{40}\\
&+ 100u_{10}^3u_{12}u_{31} - 20u_{10}^3u_{20}^2u_{42} + 80u_{10}^3u_{20}u_{21}u_{41} - 280u_{10}^3u_{20}u_{22}u_{40}\\
&- 240u_{10}^3u_{20}u_{30}u_{32} + 60u_{10}^3u_{20}u_{31}^2 + 220u_{10}^3u_{21}^2u_{40} - 700u_{10}^3u_{21}u_{22}\\
&+ 540u_{10}^3u_{21}u_{30}u_{31} - 660u_{10}^3u_{22}u_{30}^2 - 7200u_{10}^2u_{11}^2u_{31} + 700u_{10}^2u_{11}u_{12}u_{30}\\
&- 60u_{10}^2u_{11}u_{20}^2u_{41} - 240u_{10}^2u_{11}u_{20}u_{21}u_{40}\\
&+ 1500u_{10}^2u_{11}u_{20}u_{22} - 420u_{10}^2u_{11}u_{20}u_{30}u_{31}\\
&- 14800u_{10}^2u_{11}u_{21}^2 - 480u_{10}^2u_{11}u_{21}u_{30}^2\\
&+ 300u_{10}^2u_{12}u_{20}^2u_{40} + 600u_{10}^2u_{12}u_{20}u_{21}\\
&+ 900u_{10}^2u_{12}u_{20}u_{30}^2 + 60u_{10}^2u_{20}^3u_{32} - 180u_{10}^2u_{20}^2u_{21}u_{31} + 540u_{10}^2u_{20}^2u_{22}u_{30}\\
&- 420u_{10}^2u_{20}u_{21}^2u_{30} - 12000u_{10}u_{11}^3u_{30}\\
&+ 7200u_{10}u_{11}^2u_{20}u_{21} - 3000u_{10}u_{11}u_{12}u_{20}^2\\
&+ 120u_{10}u_{11}u_{20}^3u_{31} + 480u_{10}u_{11}u_{20}^2u_{21}u_{30} - 600u_{10}u_{12}u_{20}^3u_{30}\\
&- 120u_{10}u_{20}^4u_{22} + 120u_{10}u_{20}^3u_{21}^2\\
&+ 6000u_{11}^3u_{20}^2 - 120u_{11}u_{20}^4u_{21} + 120u_{12}u_{20}^5)/800
\end{aligned}
$$

3.1.4. *Third order.*

$$
\begin{aligned}
(Q_{3,4})_x &= ((Q_{2,5} + 3Q_{1,3}u_{20} - 4u_{01}u_{10}))/3\\
(Q_{3,4})_y &= (2Q_{0,7}u_{10} + Q_{1,3}u_{11} - Q_{1,8} - 2u_{01}^2 - u_{01}u_{20}^2)\\
(Q_{3,9})_x &= 3494(3(-90Q_{2,10} + 90Q_{0,7}Q_{1,3} + 120Q_{0,9}u_{10} - 180Q_{1,3}u_{01}u_{30}\\
&\quad + 870u_{01}^2u_{20} + 60u_{01}u_{10}^2u_{40} - 3000u_{01}u_{10}u_{20}u_{30} - 1670u_{01}u_{20}^3\\
&\quad + 210u_{02}u_{10}^2 + u_{20}^5))/3494\\
(Q_{3,9})_y &= 3494(5(144Q_{0,9}u_{01} - Q_{1,13} - 108Q_{1,3}u_{11}^2 + 303u_{01}^2u_{11} - 216u_{01}^2u_{20}u_{30}\\
&\quad + 492u_{01}u_{02}u_{10} + 82u_{01}u_{10}^2u_{31} + 164u_{01}u_{10}u_{11}u_{30} - 1820u_{01}u_{10}u_{20}u_{21}\\
&\quad - 694u_{01}u_{11}u_{20}^2 + 84u_{02}u_{10}^2u_{30} - 912u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} - 72u_{10}^2u_{11}u_{21}\\
&\quad + 6u_{10}^2u_{12}u_{20} - 164u_{10}u_{11}^2u_{20}))/6988\\
(Q_{3,11})_x &= 50Q_{2,12} - 3600Q_{0,7}Q_{1,3}u_{20} - 21600Q_{0,7}u_{01}u_{10} + 21600Q_{0,7}u_{10}u_{20}^2\\
&\quad + 1800Q_{0,9}Q_{1,3} - 2070Q_{0,9}u_{10}u_{20} - 7200Q_{1,11}u_{10} + 20Q_{1,3}^2u_{20}u_{40}\\
&\quad - 8414Q_{1,3}^2u_{21} + 20Q_{1,3}^2u_{30}^2 + 14328Q_{1,3}u_{01}u_{10}u_{40} - 62184Q_{1,3}u_{01}u_{11}
\end{aligned}
$$

$$
\begin{aligned}
&- 7965Q_{1,3}u_{10}^2u_{31} + 1104Q_{1,3}u_{10}u_{11}u_{30} - 29154Q_{1,3}u_{10}u_{20}u_{21}\\
&+ 32400Q_{1,6}^2 + 21600Q_{1,6}u_{01}u_{20} - 3600Q_{1,6}u_{10}^2u_{40} - 28178u_{01}^3\\
&+ 16830u_{02}u_{10}^2u_{20} + 58845u_{10}^2u_{11}^2\\
(Q_{3,11})_y &= (800Q_{2,15} + 14400Q_{0,12}Q_{1,3} - 14400Q_{0,7}^2u_{10} - 74512Q_{0,7}Q_{1,3}u_{11}\\
&+ 160Q_{0,7}Q_{1,3}u_{20}u_{30} - 85856Q_{0,7}u_{01}^2 + 28512Q_{0,7}u_{01}u_{10}u_{30}\\
&- 21616Q_{0,7}u_{01}u_{20}^2 - 21060Q_{0,7}u_{10}^2u_{21} - 2496Q_{0,7}u_{10}u_{11}u_{20}\\
&+ 40Q_{0,7}u_{20}^4 + 21600Q_{0,9}Q_{1,6} - 4140Q_{0,9}u_{10}u_{11} - 14400Q_{1,11}u_{01}\\
&- 16828Q_{1,3}^2u_{12} + 40Q_{1,3}^2u_{20}u_{31} + 40Q_{1,3}^2u_{21}u_{30} - 57056Q_{1,3}u_{01}u_{02}\\
&+ 28656Q_{1,3}u_{01}u_{10}u_{31} - 45856Q_{1,3}u_{01}u_{11}u_{30} - 21616Q_{1,3}u_{01}u_{20}u_{21}\\
&+ 160Q_{1,3}u_{01}u_{20}u_{30}^2 + 28656Q_{1,3}u_{02}u_{10}u_{30} - 10808Q_{1,3}u_{02}u_{20}^2\\
&- 15930Q_{1,3}u_{10}^2u_{22} - 58308Q_{1,3}u_{10}u_{11}u_{21} - 26448Q_{1,3}u_{10}u_{12}u_{20}\\
&+ 48064Q_{1,3}u_{11}^2u_{20} - 160Q_{1,3}u_{11}u_{20}^2u_{30}\\
&+ 80Q_{1,3}u_{20}^3u_{21} + 43200Q_{1,6}u_{01}u_{11}\\
&- 7200Q_{1,6}u_{10}^2u_{31} - 14400Q_{1,6}u_{10}u_{11}u_{30} + 14400Q_{1,6}u_{10}u_{20}u_{21}\\
&+ 7200Q_{1,6}u_{11}u_{20}^2 - 71456u_{01}^3u_{30} - 28512u_{01}^2u_{10}u_{21} + 42912u_{01}^2u_{10}u_{30}^2\\
&+ 28544u_{01}^2u_{11}u_{20} - 7216u_{01}^2u_{20}^2u_{30} - 56224u_{01}u_{02}u_{10}u_{20} - 400u_{01}u_{10}^3u_{41}\\
&+ 3600u_{01}u_{10}^2u_{11}u_{40} + 68060u_{01}u_{10}^2u_{12} - 17460u_{01}u_{10}^2u_{21}u_{30}\\
&+ 166520u_{01}u_{10}u_{11}^2 - 67008u_{01}u_{10}u_{11}u_{20}u_{30} - 28944u_{01}u_{10}u_{20}^2u_{21}\\
&+ 9568u_{01}u_{11}u_{20}^3 + 40u_{01}u_{20}^4u_{30} - 300u_{02}u_{10}^3u_{40} + 116720u_{02}u_{10}^2u_{11}\\
&+ 900u_{02}u_{10}^2u_{20}u_{30} - 10248u_{02}u_{10}u_{20}^3 + u_{10}^5u_{61}\\
&- 5u_{10}^4u_{20}u_{51} - 20u_{10}^4u_{21}u_{50}\\
&- 45u_{10}^4u_{30}u_{41} - 55u_{10}^4u_{31}u_{40} + 225u_{10}^4u_{32}\\
&- 3300u_{10}^3u_{12}u_{30} + 20u_{10}^3u_{20}^2u_{41}\\
&+ 80u_{10}^3u_{20}u_{21}u_{40} - 1700u_{10}^3u_{20}u_{22} + 140u_{10}^3u_{20}u_{30}u_{31} + 400u_{10}^3u_{21}^2\\
&+ 160u_{10}^3u_{21}u_{30}^2 - 1200u_{10}^2u_{11}^2u_{30} + 33060u_{10}^2u_{11}u_{20}u_{21} - 8430u_{10}^2u_{12}u_{20}^2\\
&- 60u_{10}^2u_{20}^3u_{31} - 240u_{10}^2u_{20}^2u_{21}u_{30} + 36096u_{10}u_{11}^2u_{20}^2 + 120u_{10}u_{20}^4u_{21}\\
&- 40u_{11}u_{20}^5)/2\\
(Q_{3,14})_x &= (-240Q_{2,15} - 4320Q_{0,12}Q_{1,3} + 10680Q_{0,12}u_{10}u_{20} + 4320Q_{0,7}^2u_{10}\\
&- 2160Q_{0,7}Q_{1,3}u_{11} - 2160Q_{0,7}u_{01}^2 + 4320Q_{0,7}u_{01}u_{10}u_{30} - 4320Q_{0,7}u_{01}u_{20}^2\\
&+ 40Q_{0,7}u_{10}^3u_{50} + 480Q_{0,7}u_{10}^2u_{20}u_{40} + 1440Q_{0,7}u_{10}^2u_{21}\\
&+ 360Q_{0,7}u_{10}^2u_{30}^2 + 2880Q_{0,7}u_{10}u_{11}u_{20} + 1440Q_{0,7}u_{10}u_{20}^2u_{30}\\
&+ 420Q_{0,7}u_{20}^4 - 1440Q_{0,9}u_{01}u_{20} - 3060Q_{0,9}u_{10}^2u_{40} - 3600Q_{0,9}u_{10}u_{11}\\
&- 10320Q_{0,9}u_{10}u_{20}u_{30} - 2460Q_{0,9}u_{20}^3\\
&- 720Q_{1,3}u_{01}u_{10}u_{31} - 1440Q_{1,3}u_{01}u_{11}u_{30}\\
&- 1080Q_{1,3}u_{02}u_{10}u_{30} + 60Q_{1,3}u_{10}^3u_{51}\\
&- 360Q_{1,3}u_{10}^2u_{21}u_{40} + 540Q_{1,3}u_{10}^2u_{22}\\
&- 540Q_{1,3}u_{10}^2u_{30}u_{31} - 11520u_{01}^2u_{10}u_{21}\\
&- 20880u_{01}u_{02}u_{10}u_{20} + 10u_{01}u_{10}^4u_{70}\\
&+ 240u_{01}u_{10}^3u_{41} - 1080u_{01}u_{10}^2u_{20}u_{31} - 20880u_{01}u_{10}u_{11}^2 + 3180u_{02}u_{10}^2u_{11}\\
&- 5040u_{02}u_{10}^2u_{20}u_{30} - 600u_{02}u_{10}u_{20}^3 + 7u_{10}^5u_{61} + 30u_{10}^4u_{21}u_{50}
\end{aligned}
$$

$$
\begin{aligned}
&- 75u_{10}^4u_{30}u_{41} - 10u_{10}^4u_{32} + 600u_{10}^3u_{11}u_{30}u_{40} + 3570u_{10}^2u_{11}^2u_{30})/60\\
(Q_{3,14})_y &= (106800Q_{0,12}u_{10}u_{11} - 21600Q_{0,7}Q_{1,3}u_{02}\\
&- 57600Q_{0,7}u_{01}u_{10}u_{21} - 28800Q_{0,7}u_{01}u_{11}u_{20}\\
&- 64800Q_{0,7}u_{02}u_{10}u_{20} + 1600Q_{0,7}u_{10}^3u_{41}\\
&+ 1200Q_{0,7}u_{10}^2u_{11}u_{40} + 36000Q_{0,7}u_{10}^2u_{12}\\
&- 3600Q_{0,7}u_{10}^2u_{21}u_{30} + 122400Q_{0,7}u_{10}u_{11}^2 + 7200Q_{0,7}u_{10}u_{11}u_{20}u_{30}\\
&+ 14400Q_{0,7}u_{10}u_{20}^2u_{21} + 2400Q_{0,7}u_{11}u_{20}^3 + 5400Q_{0,9}^2\\
&- 14400Q_{0,9}u_{01}u_{11} - 36000Q_{0,9}u_{02}u_{10}\\
&- 30600Q_{0,9}u_{10}^2u_{31} - 61200Q_{0,9}u_{10}u_{11}u_{30}\\
&- 42000Q_{0,9}u_{10}u_{20}u_{21} - 21000Q_{0,9}u_{11}u_{20}^2\\
&- 21600Q_{1,3}u_{01}u_{02}u_{30} - 7200Q_{1,3}u_{01}u_{10}u_{22} + 7200Q_{1,3}u_{01}u_{12}u_{20}\\
&- 18000Q_{1,3}u_{02}u_{10}u_{21} + 18000Q_{1,3}u_{02}u_{11}u_{20} + 600Q_{1,3}u_{10}^3u_{42}\\
&- 21600Q_{1,3}u_{01}u_{02}u_{30} - 7200Q_{1,3}u_{01}u_{10}u_{22} + 7200Q_{1,3}u_{01}u_{12}u_{20}\\
&- 18000Q_{1,3}u_{02}u_{10}u_{21} + 18000Q_{1,3}u_{02}u_{11}u_{20} + 600Q_{1,3}u_{10}^3u_{42}\\
&+ 1800Q_{1,3}u_{10}^2u_{11}u_{41} - 5400Q_{1,3}u_{10}^2u_{12}u_{40} - 1800Q_{1,3}u_{10}^2u_{20}u_{32}\\
&+ 5400Q_{1,3}u_{10}^2u_{21}u_{31} - 9000Q_{1,3}u_{10}^2u_{22}u_{30} + 18000Q_{1,3}u_{10}u_{11}u_{12}\\
&- 3600Q_{1,3}u_{10}u_{11}u_{20}u_{31}\\
&- 7200Q_{1,3}u_{10}u_{11}u_{21}u_{30} + 10800Q_{1,3}u_{10}u_{12}u_{20}u_{30}\\
&+ 3600Q_{1,3}u_{10}u_{20}^2u_{22} - 3600Q_{1,3}u_{10}u_{20}u_{21}^2 - 7200Q_{1,3}u_{11}^3\\
&+ 3600Q_{1,3}u_{11}u_{20}^2u_{21}\\
&- 3600Q_{1,3}u_{12}u_{20}^3 + 21600u_{01}^3u_{21} + 64800u_{01}^2u_{02}u_{20} - 100800u_{01}^2u_{10}u_{12}\\
&- 57600u_{01}^2u_{10}u_{21}u_{30} - 100800u_{01}^2u_{11}^2 - 28800u_{01}^2u_{11}u_{20}u_{30}\\
&+ 43200u_{01}^2u_{20}^2u_{21} - 343200u_{01}u_{02}u_{10}u_{11} - 64800u_{01}u_{02}u_{10}u_{20}u_{30}\\
&+ 28800u_{01}u_{02}u_{20}^3 + 100u_{01}u_{10}^4u_{61} + 400u_{01}u_{10}^3u_{11}u_{60}\\
&- 400u_{01}u_{10}^3u_{20}u_{51} - 400u_{01}u_{10}^3u_{21}u_{50} + 1600u_{01}u_{10}^3u_{30}u_{41}\\
&- 1200u_{01}u_{10}^2u_{11}u_{20}u_{50} + 1200u_{01}u_{10}^2u_{11}u_{30}u_{40}\\
&+ 10800u_{01}u_{10}^2u_{11}u_{31} + 103200u_{01}u_{10}^2u_{12}u_{30} + 1200u_{01}u_{10}^2u_{20}^2u_{41}\\
&+ 2400u_{01}u_{10}^2u_{20}u_{21}u_{40} - 7200u_{01}u_{10}^2u_{20}u_{22}\\
&- 7200u_{01}u_{10}^2u_{20}u_{30}u_{31} - 63600u_{01}u_{10}^2u_{21}^2 - 7200u_{01}u_{10}^2u_{21}u_{30}^2\\
&+ 144000u_{01}u_{10}u_{11}^2u_{30} + 2400u_{01}u_{10}u_{11}u_{20}^2u_{40}\\
&- 72000u_{01}u_{10}u_{11}u_{20}u_{21} - 2400u_{01}u_{10}u_{20}^3u_{31}\\
&+ 7200u_{01}u_{10}u_{20}^2u_{21}u_{30} - 28800u_{01}u_{11}^2u_{20}^2 - 1800u_{01}u_{20}^4u_{21}\\
&- 8100u_{02}^2u_{10}^2 + 100u_{02}u_{10}^4u_{60} - 400u_{02}u_{10}^3u_{20}u_{50} - 1300u_{02}u_{10}^3u_{31}\\
&- 54300u_{02}u_{10}^2u_{11}u_{30} + 1200u_{02}u_{10}^2u_{20}^2u_{40} + 6600u_{02}u_{10}^2u_{20}u_{21}\\
&- 3600u_{02}u_{10}^2u_{20}u_{30}^2 - 21000u_{02}u_{10}u_{11}u_{20}^2 - 2400u_{02}u_{10}u_{20}^3u_{30}\\
&- 360u_{02}u_{20}^5 + 73u_{10}^5u_{52} + 265u_{10}^4u_{11}u_{51} - 75u_{10}^4u_{12}u_{50} - 380u_{10}^4u_{20}u_{42}\\
&- 90u_{10}^4u_{21}u_{41} + 390u_{10}^4u_{22}u_{40} - 480u_{10}^4u_{30}u_{32} - 555u_{10}^4u_{31}^2\\
&- 400u_{10}^3u_{11}^2u_{50} - 1920u_{10}^3u_{11}u_{20}u_{41} + 2560u_{10}^3u_{11}u_{21}u_{40}\\
&+ 1100u_{10}^3u_{11}u_{22} + 2780u_{10}^3u_{11}u_{30}u_{31} + 1000u_{10}^3u_{12}u_{20}u_{40}\\
&+ 1000u_{10}^3u_{12}u_{21} + 3700u_{10}^3u_{12}u_{30}^2 + 2180u_{10}^3u_{20}^2u_{32}\\
&+ 2040u_{10}^3u_{20}u_{21}u_{31} - 1720u_{10}^3u_{20}u_{22}u_{30} - 3220u_{10}^3u_{21}^2u_{30}
\end{aligned}
$$

$$+ 2400u_{10}^2u_{11}^2u_{20}u_{40} + 77100u_{10}^2u_{11}^2u_{21} + 7200u_{10}^2u_{11}^2u_{30}^2$$
$$- 11400u_{10}^2u_{11}u_{12}u_{20} + 6540u_{10}^2u_{11}u_{20}^2u_{31} - 2160u_{10}^2u_{11}u_{20}u_{21}u_{30}$$
$$+ 20100u_{10}^2u_{12}u_{20}^2u_{30} - 8520u_{10}^2u_{20}^3u_{22} - 18660u_{10}^2u_{20}^2u_{21}^2$$
$$+ 24600u_{10}u_{11}^3u_{20} - 7200u_{10}u_{11}^2u_{20}^2u_{30} - 14640u_{10}u_{11}u_{20}^3u_{21}$$
$$+ 21000u_{10}u_{12}u_{20}^4 + 1800u_{11}^2u_{20}^4)/600$$

3.1.5. *Fourth order.*

$$\begin{aligned}
(Q_{4,8})_x &= 69120Q_{2,5}u_{01} + 229935Q_{2,5}u_{10}u_{30} + 237780Q_{2,7}u_{20}\\
&+ 177210Q_{3,4}u_{10}u_{40} - 640620Q_{3,4}u_{11}\\
&+ 1196Q_{3,9} + 5029500Q_{0,7}u_{10}^2 - 2543940Q_{1,3}Q_{1,6}\\
&+ 1703880Q_{1,3}u_{10}u_{11} + 26005830Q_{1,6}u_{10}u_{20}\\
&- 1947240Q_{1,8}u_{10}\\
(Q_{4,8})_y &= 5(-598Q_{2,12} + 56532Q_{2,5}Q_{0,7} + 56532Q_{2,5}u_{01}u_{30}\\
&+ 34173Q_{2,5}u_{10}u_{21} - 22359Q_{2,5}u_{11}u_{20}\\
&+ 47556Q_{2,7}u_{11} - 128124Q_{3,4}u_{02} + 35442Q_{3,4}u_{10}u_{31}\\
&+ 35442Q_{3,4}u_{11}u_{30} - 35442Q_{3,4}u_{20}u_{21}\\
&- 1703064Q_{0,7}u_{01}u_{10} + 70884Q_{0,7}u_{10}^2u_{30} - 70884Q_{0,7}u_{10}u_{20}^2\\
&- 106326Q_{0,9}Q_{1,3} - 102519Q_{0,9}u_{10}u_{20}\\
&- 475560Q_{1,11}u_{10} + 340776Q_{1,3}u_{02}u_{10} + 35442Q_{1,3}u_{10}u_{11}u_{30}\\
&- 35442Q_{1,3}u_{10}u_{20}u_{21} - 35442Q_{1,3}u_{11}u_{20}^2\\
&+ 5816280Q_{1,6}u_{10}u_{11} + 214236Q_{1,8}u_{01} - 35442Q_{1,8}u_{10}u_{30}\\
&+ 17721Q_{1,8}u_{20}^2 + 295118u_{01}^3 - 869916u_{01}^2u_{10}u_{30}\\
&+ 142038u_{01}^2u_{20}^2 + 170680u_{01}u_{10}^2u_{21} + 406322u_{01}u_{10}u_{11}u_{20}\\
&+ 404676u_{01}u_{10}u_{20}^2u_{30} + 21309u_{01}u_{20}^4 - 25116u_{02}u_{10}^2u_{20} + 1196u_{10}^4u_{41}\\
&- 7176u_{10}^3u_{11}u_{40} + 9568u_{10}^3u_{12} - 4784u_{10}^3u_{20}u_{31} - 16744u_{10}^3u_{21}u_{30}\\
&- 61712u_{10}^2u_{11}^2 + 21528u_{10}^2u_{11}u_{20}u_{30} + 14352u_{10}^2u_{20}^2u_{21} - 153282u_{10}u_{11}u_{20}^3)
\end{aligned}$$

3.2. **Symmetries.**

3.2.1. (x, y)*-independent.*

$$\begin{aligned}
Y_0^0 &= 1\\
Y_1^0 &= u_{10}\\
Y_4^0 &= u_{01}\\
Y_5^0 &= Q_{2,5} + 8u_{01}u_{10}\\
Y_8^0 &= -2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2\\
Y_9^0 &= 120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6} - 180Q_{1,8}u_{10} - 570u_{01}^2u_{10}\\
&+ 2070u_{01}u_{10}u_{20}^2\\
Y_{12}^0 &= -Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} + 65u_{01}^3 + 72u_{01}^2u_{10}u_{30}\\
&- 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21} + 566u_{01}u_{10}u_{11}u_{20} + 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4\\
&- 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40} + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31}\\
&- 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{21} - 48u_{10}u_{11}u_{20}^3
\end{aligned}$$

3.2.2. (x, y)*-linear.*

$$\begin{aligned}
Y_{-4}^1 &= y\\
Y_{-1}^1 &= x
\end{aligned}$$

$$
\begin{aligned}
Y^1_{0,1} &= xu_{10} - 4u\\
Y^1_{0,2} &= yu_{01} + u\\
Y^1_3 &= 4xu_{01} - Q_{1,3}\\
Y^1_{4,1} &= x(Q_{2,5} + 8u_{01}u_{10}) + 3(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01})\\
Y^1_{4,2} &= y(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) + (-3Q_{3,4} + Q_{1,3}u_{10} - 4uu_{01})\\
Y^1_7 &= (60x(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) + 3Q_{2,7}\\
&\quad + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)/60\\
Y^1_{8,1} &= 2990x(120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6}\\
&\quad - 180Q_{1,8}u_{10} - 570u_{01}^2u_{10} + 2070u_{01}u_{10}u_{20}^2)\\
&\quad + 30(162260Q_{2,5}Q_{1,3} + 170865Q_{2,5}u_{10}u_{20} + 221634Q_{2,7}u_{10}\\
&\quad - 927660Q_{3,4}u_{01} + 177210Q_{3,4}u_{10}u_{30}\\
&\quad - 88605Q_{3,4}u_{20}^2 - Q_{4,8} - 287040Q_{0,7}uu_{10} + 1162480Q_{1,3}u_{01}u_{10}\\
&\quad - 88605Q_{1,3}u_{10}u_{20}^2\\
&\quad + 14217780Q_{1,6}u_{10}^2 + 143520Q_{1,8}u - 287040uu_{01}^2 + 143520uu_{01}u_{20}^2\\
&\quad - 683460u_{01}u_{10}^2u_{20} - 73878u_{10}^2u_{20}^3)\\
Y^1_{8,2} &= 2990y(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} + 65u_{01}^3\\
&\quad + 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21} + 566u_{01}u_{10}u_{11}u_{20}\\
&\quad + 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4 - 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40}\\
&\quad + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31} - 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30}\\
&\quad + 24u_{10}^2u_{20}^2u_{21} - 48u_{10}u_{11}u_{20}^3) + (-814290Q_{2,5}Q_{1,3} - 854325Q_{2,5}u_{10}u_{20}\\
&\quad - 1102788Q_{2,7}u_{10} + 4494780Q_{3,4}u_{01} - 886050Q_{3,4}u_{10}u_{30} + 443025Q_{3,4}u_{20}^2\\
&\quad + 5Q_{4,8} + 861120Q_{0,7}uu_{10} - 5692800Q_{1,3}u_{01}u_{10}\\
&\quad + 443025Q_{1,3}u_{10}u_{20}^2 - 70765980Q_{1,6}u_{10}^2 - 430560Q_{1,8}u + 861120uu_{01}^2\\
&\quad - 430560uu_{01}u_{20}^2 + 3417300u_{01}u_{10}^2u_{20}\\
&\quad + 367596u_{10}^2u_{20}^3)\\
Y^1_{11} &= 100x(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} + 65u_{01}^3\\
&\quad + 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21} + 566u_{01}u_{10}u_{11}u_{20}\\
&\quad + 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4 - 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40}\\
&\quad + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31} - 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30}\\
&\quad + 24u_{10}^2u_{20}^2u_{21} - 48u_{10}u_{11}u_{20}^3) - 5400Q_{2,10}u_{10} - 720Q_{2,7}u_{01} + Q_{3,11}\\
&\quad + 12600Q_{0,7}Q_{1,3}u_{10} - 3600Q_{0,7}u_{10}^2u_{20} + 6435Q_{0,9}u_{10}^2 + 8414Q_{1,3}^2u_{11}\\
&\quad - 20Q_{1,3}^2u_{20}u_{30} + 1800Q_{1,3}Q_{1,8} + 7064Q_{1,3}u_{01}^2 - 14328Q_{1,3}u_{01}u_{10}u_{30}\\
&\quad + 7204Q_{1,3}u_{01}u_{20}^2 + 7965Q_{1,3}u_{10}^2u_{21} + 13224Q_{1,3}u_{10}u_{11}u_{20} - 10Q_{1,3}u_{20}^4\\
&\quad - 43200Q_{1,6}u_{01}u_{10} + 3600Q_{1,6}u_{10}^2u_{30}\\
&\quad - 3600Q_{1,6}u_{10}u_{20}^2 + 46656u_{01}^2u_{10}u_{20} - 74030u_{01}u_{10}^2u_{11} - 100800u_{01}u_{10}^2u_{20}u_{30}\\
&\quad - 48936u_{01}u_{10}u_{20}^3 + 15400u_{02}u_{10}^3 - 100u_{10}^4u_{31} + 600u_{10}^3u_{11}u_{30} + 800u_{10}^3u_{20}u_{21}\\
&\quad + 48600u_{10}^2u_{11}u_{20}^2
\end{aligned}
$$

3.2.3. *(x, y)-quadratic.*

$$
\begin{aligned}
Y^2_{-8} &= y^2\\
Y^2_{-5} &= xy
\end{aligned}
$$

$$Y^2_{-2} = x^2$$

$$Y^2_{-1} = x^2u_{10} + 4xyu_{01} - 4xu - yQ_{1,3}$$

$$Y^2_2 = 2x^2u_{01} - xQ_{1,3} - u^2_{10}$$

$$\begin{aligned} Y^2_3 &= 5x^2(-Q_{2,5} - 8u_{01}u_{10}) + 60xy(-2Q_{0,7}u_{10} + Q_{1,8} - 2u^2_{01} + u_{01}u^2_{20}) \\ &\quad + 30x(2Q_{3,4} - Q_{1,3}u_{10} + 8uu_{01}) + y(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} \\ &\quad - u_{10}u^3_{20}) + 30(-2Q_{1,3}u - u^3_{10}) \end{aligned}$$

$$\begin{aligned} Y^2_6 &= 60x^2(-2Q_{0,7}u_{10} + Q_{1,8} - 2u^2_{01} + u_{01}u^2_{20}) \\ &\quad + 2x(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u^3_{20}) \\ &\quad + 5(4Q_{2,5}u_{10} - 3Q^2_{1,3} + 8u_{01}u^2_{10}) \end{aligned}$$

$$\begin{aligned} Y^2_7 &= 2990x^2(120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u^2_{10} + 540Q_{1,3}Q_{1,6} - 180Q_{1,8}u_{10} \\ &\quad - 570u^2_{01}u_{10} + 2070u_{01}u_{10}u^2_{20}) + 29900xy(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} \\ &\quad - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} + 65u^3_{01} + 72u^2_{01}u_{10}u_{30} - 324u^2_{01}u^2_{20} \\ &\quad + 514u_{01}u^2_{10}u_{21} + 566u_{01}u_{10}u_{11}u_{20} + 72u_{01}u_{10}u^2_{20}u_{30} + 6u_{01}u^4_{20} \\ &\quad - 42u_{02}u^2_{10}u_{20} + 2u^4_{10}u_{41} - 12u^3_{10}u_{11}u_{40} + 16u^3_{10}u_{12} - 8u^3_{10}u_{20}u_{31} \\ &\quad - 28u^3_{10}u_{21}u_{30} - 257u^2_{10}u^2_{11} + 36u^2_{10}u_{11}u_{20}u_{30} + 24u^2_{10}u^2_{20}u_{21} - 48u_{10}u_{11}u^3_{20}) \\ &\quad + 10x(159270Q_{2,5}Q_{1,3} + 170865Q_{2,5}u_{10}u_{20} + 227016Q_{2,7}u_{10} - 1071180Q_{3,4}u_{01} \\ &\quad + 177210Q_{3,4}u_{10}u_{30} - 88605Q_{3,4}u^2_{20} - Q_{4,8} - 861120Q_{0,7}uu_{10} \\ &\quad + 1282080Q_{1,3}u_{01}u_{10} - 88605Q_{1,3}u_{10}u^2_{20} + 14540700Q_{1,6}u^2_{10} + 430560Q_{1,8}u \\ &\quad - 861120uu^2_{01} + 430560uu_{01}u^2_{20} - 683460u_{01}u^2_{10}u_{20} - 75672u^2_{10}u^3_{20}) \\ &\quad + 299y(-5400Q_{2,10}u_{10} - 720Q_{2,7}u_{01} + Q_{3,11} + 12600Q_{0,7}Q_{1,3}u_{10} \\ &\quad - 3600Q_{0,7}u^2_{10}u_{20} + 6435Q_{0,9}u^2_{10} + 8414Q^2_{1,3}u_{11} - 20Q^2_{1,3}u_{20}u_{30} \\ &\quad + 1800Q_{1,3}Q_{1,8} + 7064Q_{1,3}u^2_{01} - 14328Q_{1,3}u_{01}u_{10}u_{30} + 7204Q_{1,3}u_{01}u^2_{20} \\ &\quad + 7965Q_{1,3}u^2_{10}u_{21} + 13224Q_{1,3}u_{10}u_{11}u_{20} - 10Q_{1,3}u^4_{20} - 43200Q_{1,6}u_{01}u_{10} \\ &\quad + 3600Q_{1,6}u^2_{10}u_{30} - 3600Q_{1,6}u_{10}u^2_{20} + 46656u^2_{01}u_{10}u_{20} - 74030u_{01}u^2_{10}u_{11} \\ &\quad - 100800u_{01}u^2_{10}u_{20}u_{30} - 48936u_{01}u_{10}u^3_{20} + 15400u_{02}u^3_{10} - 100u^4_{10}u_{31} \\ &\quad + 600u^3_{10}u_{11}u_{30} + 800u^3_{10}u_{20}u_{21} + 48600u^2_{10}u_{11}u^2_{20}) \\ &\quad + 17940(30Q_{2,5}u^2_{10} + 12Q_{2,7}u + 60Q_{3,4}Q_{1,3} \\ &\quad - 45Q^2_{1,3}u_{10} + 240Q_{1,3}uu_{01} + 720Q_{1,6}uu_{10} - 4uu_{10}u^3_{20}) \end{aligned}$$

3.2.4. *(x, y)-cubic.*

$$Y^3_{-3} = x^3 - 2yu_{10}$$

$$Y^3_1 = 12x^3u_{01} - 9x^2Q_{1,3} - 18xu^2_{10} + 2y(-Q_{2,5} - 8u_{01}u_{10}) + 24uu_{10}$$

3.3. **Generating functions.**

3.3.1. *(x, y)-independent.*

$$G^0_0 = 1$$

$$G^0_2 = u_{20}$$

$$G^0_5 = u_{11}$$

$$G^0_6 = -18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u^3_{20}$$

$$G^0_9 = 2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u^2_{20}$$

$$G^0_{10} = -90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10}$$

$$
\begin{aligned}
&- 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11}\\
&- 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5\\
G^0_{13} &= (288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01}\\
&- 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11} + 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10}\\
&+ 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30} - 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2\\
&+ 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30} - 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20}\\
&- 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)\\
G^0_{14} &= -75600Q_{2,10}u_{01} - 37800Q_{2,10}u_{20}^2 - 700Q_{2,12}u_{20} + 33600Q_{2,5}Q_{0,7}u_{20}\\
&- 16800Q_{2,5}Q_{0,9} + 67200Q_{2,5}u_{01}u_{11} + 33600Q_{2,5}u_{01}u_{20}u_{30}\\
&- 16800Q_{2,5}u_{11}u_{20}^2 - 4200Q_{3,14} + 560Q_{3,9}u_{11} + 474600Q_{0,12}u_{10}^2\\
&+ 75600Q_{0,7}Q_{1,3}u_{01} + 113400Q_{0,7}Q_{1,3}u_{20}^2 - 302400Q_{0,7}Q_{1,6}u_{10}\\
&- 134400Q_{0,7}u_{01}u_{10}u_{20} + 2800Q_{0,7}u_{10}^3u_{40} + 478800Q_{0,7}u_{10}^2u_{11}\\
&+ 25200Q_{0,7}u_{10}^2u_{20}u_{30} + 67200Q_{0,7}u_{10}u_{20}^3 - 134400Q_{0,9}u_{01}u_{10}\\
&- 214200Q_{0,9}u_{10}^2u_{30} - 121800Q_{0,9}u_{10}u_{20}^2\\
&- 75600Q_{1,11}Q_{1,3} - 151200Q_{1,11}u_{10}u_{20}\\
&+ 700Q_{1,13}u_{10} + 302400Q_{1,3}Q_{1,6}u_{11} - 75600Q_{1,3}u_{01}^2u_{30} - 50400Q_{1,3}u_{01}u_{10}u_{21}\\
&+ 50400Q_{1,3}u_{01}u_{11}u_{20} + 75600Q_{1,3}u_{01}u_{20}^2u_{30} - 75600Q_{1,3}u_{02}u_{10}u_{20}\\
&+ 4200Q_{1,3}u_{10}^3u_{41} + 37800Q_{1,3}u_{10}^2u_{12} - 12600Q_{1,3}u_{10}^2u_{20}u_{31}\\
&- 25200Q_{1,3}u_{10}^2u_{21}u_{30} + 25200Q_{1,3}u_{10}u_{11}^2 + 25200Q_{1,3}u_{10}u_{20}^2u_{21}\\
&- 25200Q_{1,3}u_{11}u_{20}^3 + 453600Q_{1,6}^2u_{20} + 151200Q_{1,6}Q_{1,8} - 151200Q_{1,6}u_{01}^2\\
&- 12600Q_{1,6}u_{20}^4 - 50400Q_{1,8}u_{01}u_{20} - 100800Q_{1,8}u_{10}u_{11} - 8400Q_{1,8}u_{20}^3\\
&+ 599900u_{01}^3u_{20} - 1413300u_{01}^2u_{10}u_{11} - 1293600u_{01}^2u_{10}u_{20}u_{30} - 596400u_{01}^2u_{20}^3\\
&- 58800u_{01}u_{02}u_{10}^2 + 700u_{01}u_{10}^4u_{60} - 2800u_{01}u_{10}^3u_{20}u_{50} + 2800u_{01}u_{10}^3u_{30}u_{40}\\
&- 7000u_{01}u_{10}^3u_{31} + 238000u_{01}u_{10}^2u_{11}u_{30} + 8400u_{01}u_{10}^2u_{20}^2u_{40}\\
&+ 323400u_{01}u_{10}^2u_{20}u_{21} + 1629600u_{01}u_{10}u_{11}u_{20}^2 - 495600u_{01}u_{10}u_{20}^3u_{30}\\
&- 383880u_{01}u_{20}^5 - 17500u_{02}u_{10}^3u_{30} + 140700u_{02}u_{10}^2u_{20}^2\\
&+ 511u_{10}^5u_{51} - 700u_{10}^4u_{11}u_{50} - 1260u_{10}^4u_{20}u_{41} + 2380u_{10}^4u_{21}u_{40} + 1225u_{10}^4u_{22}\\
&- 3535u_{10}^4u_{30}u_{31} - 2800u_{10}^3u_{11}u_{20}u_{40} + 8400u_{10}^3u_{11}u_{21} + 16800u_{10}^3u_{11}u_{30}^2\\
&+ 5600u_{10}^3u_{12}u_{20} + 9660u_{10}^3u_{20}^2u_{31} - 33040u_{10}^3u_{20}u_{21}u_{30} + 172200u_{10}^2u_{11}^2u_{20}\\
&- 42840u_{10}^2u_{20}^3u_{21} + 260400u_{10}u_{11}u_{20}^4 + 20u_{20}^7
\end{aligned}
$$

3.3.2. (x, y)*-linear.*

$$
\begin{aligned}
G^1_{-4} &= y\\
G^1_{-1} &= x\\
G^1_{1,1} &= xu_{20} - 3u_{10}\\
G^1_{1,2} &= yu_{11} + u_{10}\\
G^1_{4} &= 4xu_{11} + 2u_{01} + u_{20}^2\\
G^1_{5,1} &= x(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad - 7Q_{2,5} - 15Q_{1,3}u_{20} - 48uu_{11} + 10u_{01}u_{10} - 9u_{10}u_{20}^2\\
G^1_{5,2} &= y(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&\quad + Q_{2,5} + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10} + u_{10}u_{20}^2
\end{aligned}
$$

$$
\begin{aligned}
G^1_8 &= 24x(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&\quad + 24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4\\
G^1_{9,1} &= 3x(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10}\\
&\quad - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11}\\
&\quad - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5)\\
&\quad - 420Q_{2,5}u_{01} - 210Q_{2,5}u_{20}^2 - 162Q_{2,7}u_{20} - 2880Q_{3,4}u_{11} - 11Q_{3,9}\\
&\quad - 2880Q_{0,7}uu_{20} + 900Q_{0,7}u_{10}^2 + 1440Q_{0,9}u - 3240Q_{1,3}Q_{1,6}\\
&\quad - 900Q_{1,3}u_{01}u_{20} + 1080Q_{1,3}u_{10}u_{11} - 150Q_{1,3}u_{20}^3 - 6480Q_{1,6}u_{10}u_{20}\\
&\quad - 1440Q_{1,8}u_{10} - 5760uu_{01}u_{11} - 2880uu_{01}u_{20}u_{30} + 1440uu_{11}u_{20}^2\\
&\quad + 10050u_{01}^2u_{10} - 25890u_{01}u_{10}u_{20}^2 + 9u_{10}u_{20}^4\\
G^1_{9,2} &= 5y(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3\\
&\quad - 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11}\\
&\quad + 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30}\\
&\quad - 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30}\\
&\quad - 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)\\
&\quad + 2(180Q_{2,5}u_{01} + 90Q_{2,5}u_{20}^2 + 72Q_{2,7}u_{20} + 1080Q_{3,4}u_{11} + 5Q_{3,9}\\
&\quad + 720Q_{0,7}uu_{20} - 540Q_{0,7}u_{10}^2 - 360Q_{0,9}u + 1620Q_{1,3}Q_{1,6}\\
&\quad + 360Q_{1,3}u_{01}u_{20} - 360Q_{1,3}u_{10}u_{11} + 60Q_{1,3}u_{20}^3 + 3240Q_{1,6}u_{10}u_{20}\\
&\quad + 720Q_{1,8}u_{10} + 1440uu_{01}u_{11} + 720uu_{01}u_{20}u_{30} - 360uu_{11}u_{20}^2\\
&\quad - 4590u_{01}^2u_{10} + 11790u_{01}u_{10}u_{20}^2 - 9u_{10}u_{20}^4)\\
G^1_{10} &= (-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20}\\
&\quad + 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20}\\
&\quad - 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2\\
&\quad + 690u_{10}u_{11}u_{20}^2 + u_{20}^5)/40
\end{aligned}
$$

3.3.3. (x, y)-*quadratic.*

$$
\begin{aligned}
G^2_{-2} &= 3x^2 - 2yu_{20}\\
G^2_0 &= x^2u_{20} + 4xyu_{11} - 2xu_{10} + y(2u_{01} + u_{20}^2) - 4u\\
G^2_3 &= 2x^2u_{11} + x(2u_{01} + u_{20}^2) - Q_{1,3} - 2u_{10}u_{20}\\
G^2_4 &= 2x^2(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + 24xy(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&\quad + 4x(-Q_{2,5} - 3Q_{1,3}u_{20} - 24uu_{11} - 2u_{01}u_{10} - 3u_{10}u_{20}^2)\\
&\quad + y(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)\\
&\quad + 12(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{01} - 2uu_{20}^2 + 3u_{10}^2u_{20})\\
G^2_7 &= 60x^2(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&\quad + 5x(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)\\
&\quad + 2(-10Q_{2,5}u_{20} - 3Q_{2,7} - 30Q_{1,3}u_{01} - 15Q_{1,3}u_{20}^2 - 20u_{01}u_{10}u_{20}\\
&\quad - 60u_{10}^2u_{11} - 9u_{10}u_{20}^3)\\
G^2_{8,1} &= (24x(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&\quad + 24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)/48\\
G^2_{8,2} &= 897x^2(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20}
\end{aligned}
$$

$$
\begin{aligned}
&+ 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20} \\
&- 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2 \\
&+ 690u_{10}u_{11}u_{20}^2 + u_{20}^5) + 2990xy(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} \\
&+ 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} \\
&- Q_{1,13} - 144Q_{1,8}u_{11} + 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} \\
&+ 20u_{01}u_{10}u_{11}u_{30} - 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} \\
&+ 12u_{02}u_{10}^2u_{30} - 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} \\
&- 42u_{11}u_{20}^4) + 598x(-60Q_{2,5}u_{01} - 30Q_{2,5}u_{20}^2 - 18Q_{2,7}u_{20} - 720Q_{3,4}u_{11} \\
&- Q_{3,9} - 1440Q_{0,7}uu_{20} - 180Q_{0,7}u_{10}^2 + 720Q_{0,9}u - 180Q_{1,3}u_{01}u_{20} \\
&+ 360Q_{1,3}u_{10}u_{11} - 30Q_{1,3}u_{20}^3 - 2880uu_{01}u_{11} - 1440uu_{01}u_{20}u_{30} \\
&+ 720uu_{11}u_{20}^2 + 870u_{01}^2u_{10} - 2310u_{01}u_{10}u_{20}^2 - 9u_{10}u_{20}^4) \\
&+ 299y(-540Q_{2,10}u_{20} - 5Q_{2,12} - 72Q_{2,7}u_{11} + 900Q_{0,7}Q_{1,3}u_{20} \\
&+ 720Q_{0,7}u_{01}u_{10} + 360Q_{0,7}u_{10}u_{20}^2 + 360Q_{0,9}Q_{1,3} + 1080Q_{0,9}u_{10}u_{20} \\
&- 360Q_{1,11}u_{10} - 1440Q_{1,3}u_{01}u_{11} - 720Q_{1,3}u_{01}u_{20}u_{30} + 360Q_{1,3}u_{11}u_{20}^2 \\
&+ 3240Q_{1,6}^2 - 2160Q_{1,6}u_{01}u_{20} - 4320Q_{1,6}u_{10}u_{11} - 360Q_{1,6}u_{20}^3 - 360Q_{1,8}u_{01} \\
&- 180Q_{1,8}u_{20}^2 - 35u_{01}^3 + 360u_{01}^2u_{10}u_{30} + 2160u_{01}^2u_{20}^2 + 2570u_{01}u_{10}^2u_{21} \\
&- 410u_{01}u_{10}u_{11}u_{20} - 9360u_{01}u_{10}u_{20}^2u_{30} - 5520u_{01}u_{20}^4 + 1410u_{02}u_{10}^2u_{20} \\
&+ 10u_{10}^4u_{41} - 60u_{10}^3u_{11}u_{40} + 80u_{10}^3u_{12} - 40u_{10}^3u_{20}u_{31} - 140u_{10}^3u_{21}u_{30} \\
&- 1285u_{10}^2u_{11}^2 + 180u_{10}^2u_{11}u_{20}u_{30} + 120u_{10}^2u_{20}^2u_{21} + 4644u_{10}u_{11}u_{20}^3 + u_{20}^6) \\
&+ 195150Q_{2,5}Q_{1,3} + 278505Q_{2,5}u_{10}u_{20} + 248544Q_{2,7}u_{10} - 855900Q_{3,4}u_{01} \\
&+ 177210Q_{3,4}u_{10}u_{30} - 196245Q_{3,4}u_{20}^2 - Q_{4,8} - 430560Q_{0,7}uu_{10} \\
&+ 26910Q_{1,3}^2u_{20} + 430560Q_{1,3}uu_{11} + 1246200Q_{1,3}u_{01}u_{10} \\
&+ 72855Q_{1,3}u_{10}u_{20}^2 + 1291680Q_{1,6}uu_{20} + 14863620Q_{1,6}u_{10}^2 \\
&+ 215280Q_{1,8}u - 215280uu_{01}^2 - 17940uu_{20}^4 - 791100u_{01}u_{10}^2u_{20} \\
&+ 215280u_{10}^3u_{11} - 29028u_{10}^2u_{20}^3
\end{aligned}
$$

$$
\begin{aligned}
G_{18,2}^2 &= 430560ux(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2) \\
&+ 17940u(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8} - 12u_{01}^2 - u_{20}^4) \\
&+ 897x^2(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20} \\
&+ 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 \\
&- 60Q_{1,8}u_{20} + 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} \\
&- 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5) \\
&+ 2990xy(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} \\
&+ 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11} \\
&+ 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30} \\
&- 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30} \\
&- 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4) \\
&+ 598x(-60Q_{2,5}u_{01} - 30Q_{2,5}u_{20}^2 - 18Q_{2,7}u_{20} - 720Q_{3,4}u_{11} - Q_{3,9} \\
&- 180Q_{0,7}u_{10}^2 - 180Q_{1,3}u_{01}u_{20} + 360Q_{1,3}u_{10}u_{11} - 30Q_{1,3}u_{20}^3 + 870u_{01}^2u_{10} \\
&- 2310u_{01}u_{10}u_{20}^2 - 9u_{10}u_{20}^4) + 299y(-540Q_{2,10}u_{20} - 5Q_{2,12} - 72Q_{2,7}u_{11} \\
&+ 900Q_{0,7}Q_{1,3}u_{20} + 720Q_{0,7}u_{01}u_{10} + 360Q_{0,7}u_{10}u_{20}^2 + 360Q_{0,9}Q_{1,3}
\end{aligned}
$$

$$
\begin{aligned}
&+ 1080Q_{0,9}u_{10}u_{20} - 360Q_{1,11}u_{10} - 1440Q_{1,3}u_{01}u_{11} - 720Q_{1,3}u_{01}u_{20}u_{30}\\
&+ 360Q_{1,3}u_{11}u_{20}^2 + 3240Q_{1,6}^2 - 2160Q_{1,6}u_{01}u_{20} - 4320Q_{1,6}u_{10}u_{11}\\
&- 360Q_{1,6}u_{20}^3 - 360Q_{1,8}u_{01} - 180Q_{1,8}u_{20}^2 - 35u_{01}^3 + 360u_{01}^2u_{10}u_{30}\\
&+ 2160u_{01}^2u_{20}^2 + 2570u_{01}u_{10}^2u_{21} - 410u_{01}u_{10}u_{11}u_{20}\\
&- 9360u_{01}u_{10}u_{20}^2u_{30} - 5520u_{01}u_{20}^4 + 1410u_{02}u_{10}^2u_{20} + 10u_{10}^4u_{41}\\
&- 60u_{10}^3u_{11}u_{40} + 80u_{10}^3u_{12} - 40u_{10}^3u_{20}u_{31} - 140u_{10}^3u_{21}u_{30} - 1285u_{10}^2u_{11}^2\\
&+ 180u_{10}^2u_{11}u_{20}u_{30} + 120u_{10}^2u_{20}^2u_{21} + 4644u_{10}u_{11}u_{20}^3 + u_{20}^6)\\
&+ 195150Q_{2,5}Q_{1,3} + 278505Q_{2,5}u_{10}u_{20} + 248544Q_{2,7}u_{10} - 855900Q_{3,4}u_{01}\\
&+ 177210Q_{3,4}u_{10}u_{30} - 196245Q_{3,4}u_{20}^2 - Q_{4,8} + 26910Q_{1,3}^2u_{20}\\
&+ 1246200Q_{1,3}u_{01}u_{10} + 72855Q_{1,3}u_{10}u_{20}^2 + 14863620Q_{1,6}u_{10}^2\\
&- 791100u_{01}u_{10}^2u_{20} + 215280u_{10}^3u_{11} - 29028u_{10}^2u_{20}^3
\end{aligned}
$$

3.3.4. (x, y)*-cubic.*

$$
\begin{aligned}
G_{-3}^3 &= 2u_{10}y - 2u_{11}y^2 - 2u_{20}xy + x^3\\
G_2^3 &= (12x^3u_{11} + 9x^2(2u_{01} + u_{20}^2) + 18x(-Q_{1,3} - 2u_{10}u_{20})\\
&\quad + 2y(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3) + 6(+4uu_{20} + u_{10}^2))/12
\end{aligned}
$$

3.4. ℓ-covering.

3.4.1. *Conservation laws and nonlocalities.*

$$
\begin{aligned}
D_x\Omega(76) &= \Omega_{yy} - \Omega_y u_{40} - \Omega u_{41}\\
D_y\Omega(76) &= -\Omega_{yy}u_{30} + 2\Omega_{xy}u_{21} - \Omega_{xx}u_{12} - 2\Omega_y u_{31} + \Omega_x u_{22} - \Omega u_{32}\\
D_x\Omega(77) &= \Omega_{yy}u_{20} - \Omega_y(u_{20}u_{40} + u_{21} + u_{30}^2)\\
&\quad + \Omega(-u_{20}u_{41} - u_{21}u_{40} + u_{22} - 2u_{30}u_{31})\\
D_y\Omega(77) &= -\Omega_{yy}u_{20}u_{30} + 2\Omega_{xy}u_{20}u_{21} - \Omega_{xx}u_{12}u_{20} - 2\Omega_y(u_{20}u_{31} + u_{21}u_{30})\\
&\quad + \Omega_x(u_{12}u_{30} + u_{20}u_{22}) + \Omega(-u_{12}u_{40} - u_{20}u_{32} - 2u_{22}u_{30})\\
D_x\Omega(78) &= \Omega_{yy}u_{11} - \Omega_y(u_{11}u_{40} + u_{12} + u_{21}u_{30}) + \Omega(-u_{11}u_{41} - 3u_{21}u_{31})\\
D_y\Omega(78) &= -\Omega_{yy}u_{11}u_{30} + 2\Omega_{xy}u_{11}u_{21} - \Omega_{xx}u_{11}u_{12} - 2\Omega_y(u_{11}u_{31} + u_{21}^2)\\
&\quad + \Omega_x(u_{11}u_{22} + u_{12}u_{21}) + \Omega(-u_{11}u_{32} - u_{12}u_{31} - 2u_{21}u_{22})\\
D_x\Omega(79) &= \Omega_{yy}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + \Omega_y(18Q_{0,7}u_{30} + 18Q_{1,6}u_{40} - 6u_{01}u_{20}u_{40} - 6u_{01}u_{21} + 12u_{01}u_{30}^2\\
&\quad - 6u_{02}u_{20} - 12u_{10}u_{11}u_{40} - 12u_{10}u_{12} - 12u_{10}u_{21}u_{30} - 3u_{11}^2\\
&\quad - 18u_{11}u_{20}u_{30} - u_{20}^3u_{40} - 3u_{20}^2u_{21} - 3u_{20}^2u_{30}^2)\\
&\quad + \Omega(54Q_{0,7}u_{31} + 18Q_{1,6}u_{41} - 6u_{01}u_{20}u_{41} - 6u_{01}u_{21}u_{40} + 6u_{01}u_{22}\\
&\quad + 42u_{01}u_{30}u_{31} + 6u_{02}u_{20}u_{40} + 48u_{02}u_{21} + 6u_{02}u_{30}^2 - 12u_{10}u_{11}u_{41}\\
&\quad - 36u_{10}u_{21}u_{31} + 3u_{11}^2u_{40} + 18u_{11}u_{12} - 54u_{11}u_{20}u_{31} + 12u_{12}u_{20}u_{30}\\
&\quad - u_{20}^3u_{41} - 3u_{20}^2u_{21}u_{40} + 3u_{20}^2u_{22} - 6u_{20}^2u_{30}u_{31}\\
&\quad - 48u_{20}u_{21}^2 - 6u_{20}u_{21}u_{30}^2)\\
D_y\Omega(79) &= \Omega_{yy}u_{30}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&\quad + 2\Omega_{xy}u_{21}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + \Omega_{xx}u_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&\quad + 2\Omega_y(18Q_{0,7}u_{21} + 18Q_{1,6}u_{31} - 6u_{01}u_{20}u_{31} + 12u_{01}u_{21}u_{30}\\
&\quad - 12u_{10}u_{11}u_{31} - 12u_{10}u_{21}^2 - 18u_{11}u_{20}u_{21} - u_{20}^3u_{31} - 3u_{20}^2u_{21}u_{30})
\end{aligned}
$$

$$
\begin{aligned}
&+ \Omega_x(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22}\\
&+ 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22})\\
&+ \Omega(36Q_{0,7}u_{22} + 18Q_{1,6}u_{32} - 6u_{01}u_{12}u_{40} - 6u_{01}u_{20}u_{32}\\
&+ 24u_{01}u_{22}u_{30} + 18u_{02}u_{12} - 12u_{10}u_{11}u_{32} - 12u_{10}u_{12}u_{31}\\
&- 24u_{10}u_{21}u_{22} - 6u_{11}u_{12}u_{30} - 36u_{11}u_{20}u_{22} - 3u_{12}u_{20}^2u_{40}\\
&- 30u_{12}u_{20}u_{21} - 6u_{12}u_{20}u_{30}^2 - u_{20}^3u_{32} - 6u_{20}^2u_{22}u_{30})\\
D_x\Omega(80) &= \Omega_{yy}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ \Omega_y(-2Q_{0,7}u_{20}u_{40} - 2Q_{0,7}u_{21} - 2Q_{0,7}u_{30}^2 + Q_{0,9}u_{40} - 4u_{01}u_{11}u_{40}\\
&- 4u_{01}u_{12} - 2u_{01}u_{20}u_{30}u_{40} - 6u_{01}u_{21}u_{30} - 2u_{01}u_{30}^3 - 2u_{02}u_{11}\\
&- 3u_{11}^2u_{30} + u_{11}u_{20}^2u_{40}) + \Omega(-2Q_{0,7}u_{20}u_{41} - 2Q_{0,7}u_{21}u_{40} + 2Q_{0,7}u_{22}\\
&- 4Q_{0,7}u_{30}u_{31} + Q_{0,9}u_{41} - 4u_{01}u_{11}u_{41} - 2u_{01}u_{20}u_{30}u_{41}\\
&- 2u_{01}u_{21}u_{30}u_{40} - 12u_{01}u_{21}u_{31} + 2u_{01}u_{22}u_{30} - 4u_{01}u_{30}^2u_{31}\\
&+ 2u_{02}u_{11}u_{40} + 6u_{02}u_{12} - 9u_{11}^2u_{31} + 4u_{11}u_{12}u_{30} + u_{11}u_{20}^2u_{41}\\
&- 16u_{11}u_{21}^2 - 2u_{11}u_{21}u_{30}^2)\\
D_y\Omega(80) &= \Omega_{yy}u_{30}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 2\Omega_{xy}u_{21}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ \Omega_{xx}u_{12}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 2\Omega_y(-2Q_{0,7}u_{20}u_{31} - 2Q_{0,7}u_{21}u_{30} + Q_{0,9}u_{31} - 4u_{01}u_{11}u_{31}\\
&- 2u_{01}u_{20}u_{30}u_{31} - 4u_{01}u_{21}^2 - 2u_{01}u_{21}u_{30}^2 - 3u_{11}^2u_{21} + u_{11}u_{20}^2u_{31})\\
&+ \Omega_x(2Q_{0,7}u_{12}u_{30} + 2Q_{0,7}u_{20}u_{22} - Q_{0,9}u_{22} + 4u_{01}u_{11}u_{22} + 4u_{01}u_{12}u_{21}\\
&+ 2u_{01}u_{12}u_{30}^2 + 2u_{01}u_{20}u_{22}u_{30} + 3u_{11}^2u_{12} - u_{11}u_{20}^2u_{22})\\
&+ \Omega(-2Q_{0,7}u_{12}u_{40} - 2Q_{0,7}u_{20}u_{32} - 4Q_{0,7}u_{22}u_{30} + Q_{0,9}u_{32}\\
&- 4u_{01}u_{11}u_{32} - 2u_{01}u_{12}u_{30}u_{40} - 4u_{01}u_{12}u_{31} - 2u_{01}u_{20}u_{30}u_{32}\\
&- 8u_{01}u_{21}u_{22} - 4u_{01}u_{22}u_{30}^2 - 2u_{02}u_{12}u_{30} - 6u_{11}^2u_{22} - 10u_{11}u_{12}u_{21}\\
&- 2u_{11}u_{12}u_{30}^2 + u_{11}u_{20}^2u_{32})\\
D_x\Omega(81) &= \Omega_{yy}(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20}\\
&+ 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20}\\
&- 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2\\
&+ 690u_{10}u_{11}u_{20}^2 + u_{20}^5) + \Omega_y(90Q_{2,10}u_{40} - 40Q_{2,5}u_{11}u_{40} - 40Q_{2,5}u_{12}\\
&- 40Q_{2,5}u_{21}u_{30} + 240Q_{0,12}u_{20} - 180Q_{0,7}^2\\
&- 270Q_{0,7}Q_{1,3}u_{40} - 360Q_{0,7}u_{01}u_{30} - 360Q_{0,7}u_{10}u_{20}u_{40} - 360Q_{0,7}u_{10}u_{21}\\
&- 360Q_{0,7}u_{10}u_{30}^2 - 60Q_{0,9}u_{10}u_{40} + 60Q_{0,9}u_{11} - 180Q_{1,11}u_{30}\\
&+ 360Q_{1,6}u_{01}u_{40} + 360Q_{1,6}u_{02} + 1080Q_{1,6}u_{11}u_{30} + 180Q_{1,6}u_{20}^2u_{40}\\
&+ 360Q_{1,6}u_{20}u_{21} + 360Q_{1,6}u_{20}u_{30}^2 + 60Q_{1,8}u_{20}u_{40} + 60Q_{1,8}u_{21}\\
&+ 60Q_{1,8}u_{30}^2 - 600u_{01}^2u_{20}u_{40} - 60u_{01}^2u_{21} - 60u_{01}^2u_{30}^2 - 120u_{01}u_{02}u_{20}\\
&+ 220u_{01}u_{10}u_{11}u_{40} - 320u_{01}u_{10}u_{12} + 1380u_{01}u_{10}u_{20}u_{30}u_{40}\\
&- 560u_{01}u_{10}u_{21}u_{30} - 240u_{01}u_{10}u_{30}^3 - 60u_{01}u_{11}^2 - 360u_{01}u_{11}u_{20}u_{30}\\
&+ 940u_{01}u_{20}^3u_{40} + 180u_{01}u_{20}^2u_{30}^2 - 270u_{02}u_{10}^2u_{40} - 240u_{02}u_{10}u_{11}\\
&- 20u_{02}u_{20}^3 - 360u_{10}u_{11}^2u_{30} - 690u_{10}u_{11}u_{20}^2u_{40} + 150u_{11}^2u_{20}^2\\
&- 60u_{11}u_{20}^3u_{30} - u_{20}^5u_{40} - 5u_{20}^4u_{21} - 5u_{20}^4u_{30}^2)\\
&+ \Omega(90Q_{2,10}u_{41} - 40Q_{2,5}u_{11}u_{41} - 120Q_{2,5}u_{21}u_{31} - 240Q_{0,12}u_{20}u_{40}
\end{aligned}
$$

$$
\begin{aligned}
&- 1920Q_{0,12}u_{21} - 240Q_{0,12}u_{30}^2 + 180Q_{0,7}^2u_{40} - 270Q_{0,7}Q_{1,3}u_{41}\\
&+ 360Q_{0,7}u_{01}u_{30}u_{40} - 360Q_{0,7}u_{10}u_{20}u_{41} - 360Q_{0,7}u_{10}u_{21}u_{40}\\
&+ 360Q_{0,7}u_{10}u_{22} - 720Q_{0,7}u_{10}u_{30}u_{31} + 2880Q_{0,7}u_{11}u_{21}\\
&+ 360Q_{0,7}u_{11}u_{30}^2 - 60Q_{0,9}u_{10}u_{41} - 60Q_{0,9}u_{11}u_{40} - 180Q_{0,9}u_{12}\\
&- 540Q_{1,11}u_{31} + 360Q_{1,6}u_{01}u_{41} - 360Q_{1,6}u_{02}u_{40} + 3240Q_{1,6}u_{11}u_{31}\\
&- 720Q_{1,6}u_{12}u_{30} + 180Q_{1,6}u_{20}^2u_{41} + 360Q_{1,6}u_{20}u_{21}u_{40}\\
&- 360Q_{1,6}u_{20}u_{22} + 720Q_{1,6}u_{20}u_{30}u_{31} + 2880Q_{1,6}u_{21}^2 + 360Q_{1,6}u_{21}u_{30}^2\\
&+ 60Q_{1,8}u_{20}u_{41} + 60Q_{1,8}u_{21}u_{40} - 60Q_{1,8}u_{22} + 120Q_{1,8}u_{30}u_{31}\\
&- 600u_{01}^2u_{20}u_{41} - 60u_{01}^2u_{21}u_{40} + 60u_{01}^2u_{22} + 180u_{01}^2u_{30}^2u_{40}\\
&+ 420u_{01}^2u_{30}u_{31} + 120u_{01}u_{02}u_{20}u_{40} + 960u_{01}u_{02}u_{21} + 120u_{01}u_{02}u_{30}^2\\
&+ 220u_{01}u_{10}u_{11}u_{41} + 1380u_{01}u_{10}u_{20}u_{30}u_{41} - 240u_{01}u_{10}u_{21}u_{30}u_{40}\\
&- 960u_{01}u_{10}u_{21}u_{31} + 240u_{01}u_{10}u_{22}u_{30} - 480u_{01}u_{10}u_{30}^2u_{31}\\
&+ 60u_{01}u_{11}^2u_{40} + 360u_{01}u_{11}u_{12} - 1080u_{01}u_{11}u_{20}u_{31}\\
&+ 2880u_{01}u_{11}u_{21}u_{30} + 360u_{01}u_{11}u_{30}^3 + 240u_{01}u_{12}u_{20}u_{30} + 940u_{01}u_{20}^3u_{41}\\
&+ 540u_{01}u_{20}^2u_{30}u_{31} - 960u_{01}u_{20}u_{21}^2 - 120u_{01}u_{20}u_{21}u_{30}^2 - 270u_{02}u_{10}^2u_{41}\\
&+ 240u_{02}u_{10}u_{11}u_{40} + 720u_{02}u_{10}u_{12} + 20u_{02}u_{20}^3u_{40} + 480u_{02}u_{20}^2u_{21}\\
&+ 60u_{02}u_{20}^2u_{30}^2 - 1080u_{10}u_{11}^2u_{31} + 480u_{10}u_{11}u_{12}u_{30} - 690u_{10}u_{11}u_{20}^2u_{41}\\
&- 1920u_{10}u_{11}u_{21}^2 - 240u_{10}u_{11}u_{21}u_{30}^2 - 150u_{11}^2u_{20}^2u_{40} - 2400u_{11}^2u_{20}u_{21}\\
&- 300u_{11}^2u_{20}u_{30}^2 - 180u_{11}u_{20}^3u_{31} + 40u_{12}u_{20}^3u_{30} - u_{20}^5u_{41} - 5u_{20}^4u_{21}u_{40}\\
&+ 5u_{20}^4u_{22} - 10u_{20}^4u_{30}u_{31} - 160u_{20}^3u_{21}^2 - 20u_{20}^3u_{21}u_{30}^2)
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(81) &= \Omega_{yy}u_{30}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20}\\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2\\
&+ 60Q_{1,8}u_{20} - 600u_{01}^2u_{20}\\
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2\\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 2\Omega_{xy}u_{21}(-90Q_{2,10} + 40Q_{2,5}u_{11}\\
&+ 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01}\\
&- 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11}\\
&- 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5)\\
&+ \Omega_{xx}u_{12}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20}\\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2 + 60Q_{1,8}u_{20} - 600u_{01}^2u_{20}\\
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2\\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 2\Omega_y(90Q_{2,10}u_{31} - 40Q_{2,5}u_{11}u_{31} - 40Q_{2,5}u_{21}^2\\
&- 270Q_{0,7}Q_{1,3}u_{31} - 360Q_{0,7}u_{10}u_{20}u_{31} - 360Q_{0,7}u_{10}u_{21}u_{30}\\
&- 60Q_{0,9}u_{10}u_{31} - 180Q_{1,11}u_{21} + 360Q_{1,6}u_{01}u_{31} + 1080Q_{1,6}u_{11}u_{21}\\
&+ 180Q_{1,6}u_{20}^2u_{31} + 360Q_{1,6}u_{20}u_{21}u_{30} + 60Q_{1,8}u_{20}u_{31} + 60Q_{1,8}u_{21}u_{30}\\
&- 600u_{01}^2u_{20}u_{31} + 120u_{01}^2u_{21}u_{30} + 220u_{01}u_{10}u_{11}u_{31}\\
&+ 1380u_{01}u_{10}u_{20}u_{30}u_{31} - 320u_{01}u_{10}u_{21}^2 - 240u_{01}u_{10}u_{21}u_{30}^2\\
&- 360u_{01}u_{11}u_{20}u_{21} + 940u_{01}u_{20}^3u_{31} + 180u_{01}u_{20}^2u_{21}u_{30} - 270u_{02}u_{10}^2u_{31}\\
&- 360u_{10}u_{11}^2u_{21} - 690u_{10}u_{11}u_{20}^2u_{31} - 60u_{11}u_{20}^3u_{21} - u_{20}^5u_{31}\\
&- 5u_{20}^4u_{21}u_{30}) + \Omega_x(-90Q_{2,10}u_{22} + 40Q_{2,5}u_{11}u_{22} + 40Q_{2,5}u_{12}u_{21}\\
&+ 270Q_{0,7}Q_{1,3}u_{22} + 360Q_{0,7}u_{10}u_{12}u_{30} + 360Q_{0,7}u_{10}u_{20}u_{22}
\end{aligned}
$$

$$+ 60Q_{0,9}u_{10}u_{22} + 180Q_{1,11}u_{12} - 360Q_{1,6}u_{01}u_{22} - 1080Q_{1,6}u_{11}u_{12}$$
$$- 360Q_{1,6}u_{12}u_{20}u_{30} - 180Q_{1,6}u_{20}^2u_{22} - 60Q_{1,8}u_{12}u_{30} - 60Q_{1,8}u_{20}u_{22}$$
$$- 120u_{01}^2u_{12}u_{30} + 600u_{01}^2u_{20}u_{22} - 220u_{01}u_{10}u_{11}u_{22} + 320u_{01}u_{10}u_{12}u_{21}$$
$$+ 240u_{01}u_{10}u_{12}u_{30}^2 - 1380u_{01}u_{10}u_{20}u_{22}u_{30} + 360u_{01}u_{11}u_{12}u_{20}$$
$$- 180u_{01}u_{12}u_{20}^2u_{30} - 940u_{01}u_{20}^3u_{22} + 270u_{02}u_{10}^2u_{22} + 360u_{10}u_{11}^2u_{12}$$
$$+ 690u_{10}u_{11}u_{20}^2u_{22} + 60u_{11}u_{12}u_{20}^3 + 5u_{12}u_{20}^4u_{30} + u_{20}^5u_{22})$$
$$+ \Omega(90Q_{2,10}u_{32} - 40Q_{2,5}u_{11}u_{32} - 40Q_{2,5}u_{12}u_{31} - 80Q_{2,5}u_{21}u_{22}$$
$$- 720Q_{0,12}u_{12} - 270Q_{0,7}Q_{1,3}u_{32} - 360Q_{0,7}u_{10}u_{12}u_{40} - 360Q_{0,7}u_{10}u_{20}u_{32}$$
$$- 720Q_{0,7}u_{10}u_{22}u_{30} + 1080Q_{0,7}u_{11}u_{12} - 60Q_{0,9}u_{10}u_{32} + 60Q_{0,9}u_{12}u_{30}$$
$$- 360Q_{1,11}u_{22} + 360Q_{1,6}u_{01}u_{32} + 2160Q_{1,6}u_{11}u_{22} + 360Q_{1,6}u_{12}u_{20}u_{40}$$
$$+ 1800Q_{1,6}u_{12}u_{21} + 360Q_{1,6}u_{12}u_{30}^2 + 180Q_{1,6}u_{20}^2u_{32} + 720Q_{1,6}u_{20}u_{22}u_{30}$$
$$+ 60Q_{1,8}u_{12}u_{40} + 60Q_{1,8}u_{20}u_{32} + 120Q_{1,8}u_{22}u_{30}$$
$$- 60u_{01}^2u_{12}u_{40} - 600u_{01}^2u_{20}u_{32}$$
$$+ 240u_{01}^2u_{22}u_{30} + 360u_{01}u_{02}u_{12} + 220u_{01}u_{10}u_{11}u_{32} - 240u_{01}u_{10}u_{12}u_{30}u_{40}$$
$$- 320u_{01}u_{10}u_{12}u_{31} + 1380u_{01}u_{10}u_{20}u_{30}u_{32} - 640u_{01}u_{10}u_{21}u_{22}$$
$$- 480u_{01}u_{10}u_{22}u_{30}^2 + 960u_{01}u_{11}u_{12}u_{30}$$
$$- 720u_{01}u_{11}u_{20}u_{22} - 600u_{01}u_{12}u_{20}u_{21}$$
$$- 120u_{01}u_{12}u_{20}u_{30}^2 + 940u_{01}u_{20}^3u_{32} + 360u_{01}u_{20}^2u_{22}u_{30} - 270u_{02}u_{10}^2u_{32}$$
$$- 240u_{02}u_{10}u_{12}u_{30} + 180u_{02}u_{12}u_{20}^2 - 720u_{10}u_{11}^2u_{22} - 1200u_{10}u_{11}u_{12}u_{21}$$
$$- 240u_{10}u_{11}u_{12}u_{30}^2 - 690u_{10}u_{11}u_{20}^2u_{32} - 900u_{11}^2u_{12}u_{20} - 120u_{11}u_{20}^3u_{22}$$
$$- 5u_{12}u_{20}^4u_{40} - 100u_{12}u_{20}^3u_{21} - 20u_{12}u_{20}^3u_{30}^2 - u_{20}^5u_{32} - 10u_{20}^4u_{22}u_{30})$$

$$D_x\Omega(83) = \Omega_y - \Omega u_{40}$$
$$D_y\Omega(83) = \Omega(76)$$
$$D_x\Omega(84) = \Omega(76) + \Omega_y u_{30} + 3\Omega u_{31}$$
$$D_y\Omega(84) = 2\Omega_y u_{21} - \Omega_x u_{12} + 2\Omega u_{22}$$
$$D_x\Omega(85) = \Omega(77) + 3\Omega_{yy}u_{10} + \Omega_y(-3u_{10}u_{40} - 3u_{11} - 2u_{20}u_{30})$$
$$+ 3\Omega(-u_{10}u_{41} + u_{11}u_{40} + u_{12} - 2u_{20}u_{31})$$
$$D_y\Omega(85) = -3\Omega_{yy}u_{10}u_{30} + 6\Omega_{xy}u_{10}u_{21} - 3\Omega_{xx}u_{10}u_{12} + 2\Omega_y(-3u_{10}u_{31} - 2u_{20}u_{21})$$
$$+ \Omega_x(3u_{10}u_{22} + 2u_{12}u_{20}) + \Omega(-3u_{10}u_{32} - u_{12}u_{30} - 4u_{20}u_{22})$$
$$D_x\Omega(86) = -\Omega_{yy}u_{10} + \Omega_y(u_{10}u_{40} + 2u_{11} + u_{20}u_{30})$$
$$+ \Omega(u_{10}u_{41} - 2u_{11}u_{40} - 3u_{12} + 3u_{20}u_{31})$$
$$D_y\Omega(86) = \Omega(78) + \Omega_{yy}u_{10}u_{30} - 2\Omega_{xy}u_{10}u_{21} + \Omega_{xx}u_{10}u_{12}$$
$$+ 2\Omega_y(u_{10}u_{31} + u_{20}u_{21}) - \Omega_x(u_{10}u_{22}$$
$$+ u_{12}u_{20}) + \Omega(u_{10}u_{32} + u_{12}u_{30} + 2u_{20}u_{22})$$
$$D_x\Omega(87) = (4\Omega(78) + \Omega_{yy}(-2u_{01} - u_{20}^2)$$
$$+ \Omega_y(2u_{01}u_{40} + 2u_{02} + 6u_{11}u_{30} + u_{20}^2u_{40} + 2u_{20}u_{21} + 2u_{20}u_{30}^2)$$
$$+ \Omega(2u_{01}u_{41} - 2u_{02}u_{40} + 18u_{11}u_{31} - 4u_{12}u_{30} + u_{20}^2u_{41} + 2u_{20}u_{21}u_{40}$$
$$- 2u_{20}u_{22} + 4u_{20}u_{30}u_{31} + 16u_{21}^2 + 2u_{21}u_{30}^2))/4$$
$$D_y\Omega(87) = (\Omega_{yy}u_{30}(2u_{01} + u_{20}^2) + 2\Omega_{xy}u_{21}(-2u_{01} - u_{20}^2) + \Omega_{xx}u_{12}(2u_{01} + u_{20}^2)$$
$$+ 2\Omega_y(2u_{01}u_{31} + 6u_{11}u_{21} + u_{20}^2u_{31} + 2u_{20}u_{21}u_{30})$$
$$+ \Omega_x(-2u_{01}u_{22} - 6u_{11}u_{12} - 2u_{12}u_{20}u_{30} - u_{20}^2u_{22})$$
$$+ \Omega(2u_{01}u_{32} + 12u_{11}u_{22} + 2u_{12}u_{20}u_{40}$$

$$
\begin{aligned}
&+ 10u_{12}u_{21} + 2u_{12}u_{30}^2 + u_{20}^2u_{32} + 4u_{20}u_{22}u_{30}))/4\\
D_x\Omega(88) &= \Omega(79) + \Omega_{yy}(7Q_{2,5} + 15Q_{1,3}u_{20} + 48uu_{11} - 10u_{01}u_{10} + 9u_{10}u_{20}^2)\\
&+ \Omega_y(-7Q_{2,5}u_{40} - 30Q_{0,7}u_{20} + 21Q_{0,9} - 15Q_{1,3}u_{20}u_{40} - 15Q_{1,3}u_{21}\\
&- 15Q_{1,3}u_{30}^2 + 108Q_{1,6}u_{30} - 48uu_{11}u_{40} - 48uu_{12} - 48uu_{21}u_{30}\\
&+ 10u_{01}u_{10}u_{40} - 24u_{01}u_{11} - 30u_{01}u_{20}u_{30} - 18u_{02}u_{10} - 54u_{10}u_{11}u_{30}\\
&- 9u_{10}u_{20}^2u_{40} - 18u_{10}u_{20}u_{21} - 18u_{10}u_{20}u_{30}^2 + 21u_{11}u_{20}^2)\\
&+ \Omega(-7Q_{2,5}u_{41} + 30Q_{0,7}u_{20}u_{40} + 240Q_{0,7}u_{21} + 30Q_{0,7}u_{30}^2\\
&- 21Q_{0,9}u_{40} - 15Q_{1,3}u_{20}u_{41} - 15Q_{1,3}u_{21}u_{40} + 15Q_{1,3}u_{22}\\
&- 30Q_{1,3}u_{30}u_{31} + 324Q_{1,6}u_{31} - 48uu_{11}u_{41} - 144uu_{21}u_{31}\\
&+ 10u_{01}u_{10}u_{41} + 24u_{01}u_{11}u_{40} + 72u_{01}u_{12} + 30u_{01}u_{20}u_{30}u_{40}\\
&+ 240u_{01}u_{21}u_{30} + 30u_{01}u_{30}^3 + 18u_{02}u_{10}u_{40} - 162u_{10}u_{11}u_{31}\\
&+ 36u_{10}u_{12}u_{30} - 9u_{10}u_{20}^2u_{41} - 18u_{10}u_{20}u_{21}u_{40} + 18u_{10}u_{20}u_{22}\\
&- 36u_{10}u_{20}u_{30}u_{31} - 144u_{10}u_{21}^2 - 18u_{10}u_{21}u_{30}^2 - 21u_{11}u_{20}^2u_{40}\\
&- 96u_{11}u_{20}u_{21} - 12u_{11}u_{20}u_{30}^2)\\
D_y\Omega(88) &= \Omega_{yy}u_{30}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 48uu_{11} + 10u_{01}u_{10} - 9u_{10}u_{20}^2)\\
&+ 2\Omega_{xy}u_{21}(7Q_{2,5} + 15Q_{1,3}u_{20} + 48uu_{11} - 10u_{01}u_{10} + 9u_{10}u_{20}^2)\\
&+ \Omega_{xx}u_{12}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 48uu_{11} + 10u_{01}u_{10} - 9u_{10}u_{20}^2)\\
&+ 2\Omega_y(-7Q_{2,5}u_{31} - 15Q_{1,3}u_{20}u_{31} - 15Q_{1,3}u_{21}u_{30} + 108Q_{1,6}u_{21}\\
&- 48uu_{11}u_{31} - 48uu_{21}^2 + 10u_{01}u_{10}u_{31} - 54u_{10}u_{11}u_{21} - 9u_{10}u_{20}^2u_{31}\\
&- 18u_{10}u_{20}u_{21}u_{30}) + \Omega_x(7Q_{2,5}u_{22} + 15Q_{1,3}u_{12}u_{30} + 15Q_{1,3}u_{20}u_{22}\\
&- 108Q_{1,6}u_{12} + 48uu_{11}u_{22} + 48uu_{12}u_{21} - 10u_{01}u_{10}u_{22} + 54u_{10}u_{11}u_{12}\\
&+ 18u_{10}u_{12}u_{20}u_{30} + 9u_{10}u_{20}^2u_{22})\\
&+ \Omega(-7Q_{2,5}u_{32} + 90Q_{0,7}u_{12} - 15Q_{1,3}u_{12}u_{40} - 15Q_{1,3}u_{20}u_{32}\\
&- 30Q_{1,3}u_{22}u_{30} + 216Q_{1,6}u_{22} - 48uu_{11}u_{32} - 48uu_{12}u_{31} - 96uu_{21}u_{22}\\
&+ 10u_{01}u_{10}u_{32} + 66u_{01}u_{12}u_{30} - 108u_{10}u_{11}u_{22} - 18u_{10}u_{12}u_{20}u_{40}\\
&- 90u_{10}u_{12}u_{21} - 18u_{10}u_{12}u_{30}^2 - 9u_{10}u_{20}^2u_{32}\\
&- 36u_{10}u_{20}u_{22}u_{30} - 36u_{11}u_{12}u_{20})\\
D_x\Omega(89) &= \Omega_{yy}(-Q_{2,5} - 2Q_{1,3}u_{20} - 4uu_{11} + 2u_{01}u_{10} - u_{10}u_{20}^2)\\
&+ \Omega_y(Q_{2,5}u_{40} + 6Q_{0,7}u_{20} - 4Q_{0,9} + 2Q_{1,3}u_{20}u_{40} + 2Q_{1,3}u_{21}\\
&+ 2Q_{1,3}u_{30}^2 - 18Q_{1,6}u_{30} + 4uu_{11}u_{40} + 4uu_{12} + 4uu_{21}u_{30} - 2u_{01}u_{10}u_{40}\\
&+ 4u_{01}u_{11} + 6u_{01}u_{20}u_{30} + 2u_{02}u_{10} + 6u_{10}u_{11}u_{30} + u_{10}u_{20}^2u_{40}\\
&+ 2u_{10}u_{20}u_{21} + 2u_{10}u_{20}u_{30}^2 - 4u_{11}u_{20}^2) + \Omega(Q_{2,5}u_{41} - 6Q_{0,7}u_{20}u_{40}\\
&- 48Q_{0,7}u_{21} - 6Q_{0,7}u_{30}^2 + 4Q_{0,9}u_{40} + 2Q_{1,3}u_{20}u_{41} + 2Q_{1,3}u_{21}u_{40}\\
&- 2Q_{1,3}u_{22} + 4Q_{1,3}u_{30}u_{31} - 54Q_{1,6}u_{31} + 4uu_{11}u_{41} + 12uu_{21}u_{31}\\
&- 2u_{01}u_{10}u_{41} - 4u_{01}u_{11}u_{40} - 12u_{01}u_{12} - 6u_{01}u_{20}u_{30}u_{40}\\
&- 48u_{01}u_{21}u_{30} - 6u_{01}u_{30}^3 - 2u_{02}u_{10}u_{40} + 18u_{10}u_{11}u_{31}\\
&- 4u_{10}u_{12}u_{30} + u_{10}u_{20}^2u_{41} + 2u_{10}u_{20}u_{21}u_{40} - 2u_{10}u_{20}u_{22}\\
&+ 4u_{10}u_{20}u_{30}u_{31} + 16u_{10}u_{21}^2 + 2u_{10}u_{21}u_{30}^2 + 4u_{11}u_{20}^2u_{40}\\
&+ 16u_{11}u_{20}u_{21} + 2u_{11}u_{20}u_{30}^2)\\
D_y\Omega(89) &= \Omega(80) + \Omega_{yy}u_{30}(Q_{2,5} + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10} + u_{10}u_{20}^2)\\
&+ 2\Omega_{xy}u_{21}(-Q_{2,5} - 2Q_{1,3}u_{20} - 4uu_{11} + 2u_{01}u_{10} - u_{10}u_{20}^2)\\
&+ \Omega_{xx}u_{12}(Q_{2,5} + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10} + u_{10}u_{20}^2)
\end{aligned}
$$

$$
\begin{aligned}
&+ 2\Omega_y(Q_{2,5}u_{31} + 2Q_{1,3}u_{20}u_{31} + 2Q_{1,3}u_{21}u_{30} - 18Q_{1,6}u_{21} + 4uu_{11}u_{31}\\
&+ 4uu_{21}^2 - 2u_{01}u_{10}u_{31} + 6u_{10}u_{11}u_{21} + u_{10}u_{20}^2u_{31} + 2u_{10}u_{20}u_{21}u_{30})\\
&+ \Omega_x(-Q_{2,5}u_{22} - 2Q_{1,3}u_{12}u_{30} - 2Q_{1,3}u_{20}u_{22} + 18Q_{1,6}u_{12}\\
&- 4uu_{11}u_{22} - 4uu_{12}u_{21} + 2u_{01}u_{10}u_{22} - 6u_{10}u_{11}u_{12}\\
&- 2u_{10}u_{12}u_{20}u_{30} - u_{10}u_{20}^2u_{22}) + \Omega(Q_{2,5}u_{32} - 18Q_{0,7}u_{12}\\
&+ 2Q_{1,3}u_{12}u_{40} + 2Q_{1,3}u_{20}u_{32} + 4Q_{1,3}u_{22}u_{30} - 36Q_{1,6}u_{22}\\
&+ 4uu_{11}u_{32} + 4uu_{12}u_{31} + 8uu_{21}u_{22} - 2u_{01}u_{10}u_{32} - 14u_{01}u_{12}u_{30}\\
&+ 12u_{10}u_{11}u_{22} + 2u_{10}u_{12}u_{20}u_{40} + 10u_{10}u_{12}u_{21} + 2u_{10}u_{12}u_{30}^2\\
&+ u_{10}u_{20}^2u_{32} + 4u_{10}u_{20}u_{22}u_{30} + 6u_{11}u_{12}u_{20})
\end{aligned}
$$

$$
\begin{aligned}
D_x\Omega(90) &= (24\Omega(80) + \Omega_{yy}(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8}\\
&- 12u_{01}^2 - u_{20}^4) + \Omega_y(-48Q_{0,12} + 24Q_{0,7}u_{10}u_{40} - 24Q_{0,7}u_{11}\\
&- 36Q_{0,9}u_{30} - 24Q_{1,3}u_{11}u_{40} - 24Q_{1,3}u_{12} - 24Q_{1,3}u_{21}u_{30}\\
&- 72Q_{1,6}u_{20}u_{40} - 72Q_{1,6}u_{21} - 72Q_{1,6}u_{30}^2 - 12Q_{1,8}u_{40} + 12u_{01}^2u_{40}\\
&+ 24u_{01}u_{02} + 48u_{01}u_{11}u_{30} + 24u_{01}u_{20}u_{21} + 24u_{01}u_{20}u_{30}^2\\
&+ 12u_{02}u_{20}^2 - 12u_{11}^2u_{20} + u_{20}^4u_{40} + 4u_{20}^3u_{21} + 4u_{20}^3u_{30}^2)\\
&+ \Omega(48Q_{0,12}u_{40} + 24Q_{0,7}u_{10}u_{41} + 24Q_{0,7}u_{11}u_{40} + 72Q_{0,7}u_{12}\\
&- 108Q_{0,9}u_{31} - 24Q_{1,3}u_{11}u_{41} - 72Q_{1,3}u_{21}u_{31} - 72Q_{1,6}u_{20}u_{41}\\
&- 72Q_{1,6}u_{21}u_{40} + 72Q_{1,6}u_{22} - 144Q_{1,6}u_{30}u_{31} - 12Q_{1,8}u_{41}\\
&+ 12u_{01}^2u_{41} - 24u_{01}u_{02}u_{40} + 24u_{01}u_{11}u_{30}u_{40} + 216u_{01}u_{11}u_{31}\\
&+ 24u_{01}u_{12}u_{30} + 24u_{01}u_{20}u_{21}u_{40} - 24u_{01}u_{20}u_{22} + 48u_{01}u_{20}u_{30}u_{31}\\
&+ 192u_{01}u_{21}^2 + 24u_{01}u_{21}u_{30}^2 - 12u_{02}u_{20}^2u_{40} - 192u_{02}u_{20}u_{21}\\
&- 24u_{02}u_{20}u_{30}^2 + 12u_{11}^2u_{20}u_{40} + 288u_{11}^2u_{21} + 36u_{11}^2u_{30}^2 - 72u_{11}u_{12}u_{20}\\
&- 24u_{12}u_{20}^2u_{30} + u_{20}^4u_{41} + 4u_{20}^3u_{21}u_{40} - 4u_{20}^3u_{22} + 8u_{20}^3u_{30}u_{31}\\
&+ 96u_{20}^2u_{21}^2 + 12u_{20}^2u_{21}u_{30}^2))/24
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(90) &= (\Omega_{yy}u_{30}(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)\\
&+ 2\Omega_{xy}u_{21}(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8}\\
&- 12u_{01}^2 - u_{20}^4) + \Omega_{xx}u_{12}(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8}\\
&+ 12u_{01}^2 + u_{20}^4) + 2\Omega_y(24Q_{0,7}u_{10}u_{31} - 36Q_{0,9}u_{21} - 24Q_{1,3}u_{11}u_{31}\\
&- 24Q_{1,3}u_{21}^2 - 72Q_{1,6}u_{20}u_{31} - 72Q_{1,6}u_{21}u_{30} - 12Q_{1,8}u_{31} + 12u_{01}^2u_{31}\\
&+ 72u_{01}u_{11}u_{21} + 24u_{01}u_{20}u_{21}u_{30} + u_{20}^4u_{31} + 4u_{20}^3u_{21}u_{30})\\
&+ \Omega_x(-24Q_{0,7}u_{10}u_{22} + 36Q_{0,9}u_{12} + 24Q_{1,3}u_{11}u_{22} + 24Q_{1,3}u_{12}u_{21}\\
&+ 72Q_{1,6}u_{12}u_{30} + 72Q_{1,6}u_{20}u_{22} + 12Q_{1,8}u_{22} - 12u_{01}^2u_{22}\\
&- 72u_{01}u_{11}u_{12} - 24u_{01}u_{12}u_{20}u_{30} - 4u_{12}u_{20}^3u_{30} - u_{20}^4u_{22})\\
&+ \Omega(24Q_{0,7}u_{10}u_{32} - 24Q_{0,7}u_{12}u_{30} - 72Q_{0,9}u_{22} - 24Q_{1,3}u_{11}u_{32}\\
&- 24Q_{1,3}u_{12}u_{31} - 48Q_{1,3}u_{21}u_{22} - 72Q_{1,6}u_{12}u_{40} - 72Q_{1,6}u_{20}u_{32}\\
&- 144Q_{1,6}u_{22}u_{30} - 12Q_{1,8}u_{32} + 12u_{01}^2u_{32} + 144u_{01}u_{11}u_{22}\\
&+ 24u_{01}u_{12}u_{20}u_{40} + 120u_{01}u_{12}u_{21} + 48u_{01}u_{20}u_{22}u_{30} - 72u_{02}u_{12}u_{20}\\
&+ 108u_{11}^2u_{12} + 24u_{11}u_{12}u_{20}u_{30} + 4u_{12}u_{20}^3u_{40} + 60u_{12}u_{20}^2u_{21}\\
&+ 12u_{12}u_{20}^2u_{30}^2 + u_{20}^4u_{32} + 8u_{20}^3u_{22}u_{30}))/24
\end{aligned}
$$

$$
\begin{aligned}
D_x\Omega(91) &= 3\Omega(84) + \Omega_y u_{20} + \Omega(-u_{20}u_{40} - 8u_{21} - u_{30}^2)\\
D_y\Omega(91) &= \Omega(77) - 3\Omega u_{12}\\
D_x\Omega(92) &= (4\Omega(86) + 2\Omega(85) - 4\Omega_{yy}u + \Omega_y(4uu_{40} + 2u_{01} + 6u_{10}u_{30} - u_{20}^2)
\end{aligned}
$$

$$
\begin{aligned}
&+ \Omega(4uu_{41} - 2u_{01}u_{40} + 18u_{10}u_{31} + u_{20}^2u_{40} + 16u_{20}u_{21} + 2u_{20}u_{30}^2))/2\\
D_y\Omega(92) &= 2\Omega(87) + 2\Omega_{yy}uu_{30} - 4\Omega_{xy}uu_{21} + 2\Omega_{xx}uu_{12}\\
&+ 2\Omega_y(2uu_{31} + 3u_{10}u_{21})\\
&+ \Omega_x(-2uu_{22} - 3u_{10}u_{12}) + \Omega(2uu_{32} + 6u_{10}u_{22} + 3u_{12}u_{20})\\
D_x\Omega(93) &= (24\Omega(89) + 4\Omega(88) + 12\Omega_{yy}(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{01}\\
&- 2uu_{20}^2 + 3u_{10}^2u_{20}) + \Omega_y(12Q_{2,5}u_{30} + 24Q_{3,4}u_{40} - 48Q_{0,7}u_{10}\\
&- 36Q_{1,3}u_{10}u_{40} + 12Q_{1,3}u_{11} + 72Q_{1,6}u_{20} - 12Q_{1,8} + 48uu_{01}u_{40}\\
&+ 48uu_{02} + 144uu_{11}u_{30} + 24uu_{20}^2u_{40} + 48uu_{20}u_{21} + 48uu_{20}u_{30}^2 - 12u_{01}^2\\
&- 120u_{01}u_{10}u_{30} - 36u_{10}^2u_{20}u_{40} - 36u_{10}^2u_{21} - 36u_{10}^2u_{30}^2 - u_{20}^4)\\
&+ \Omega(36Q_{2,5}u_{31} + 24Q_{3,4}u_{41} + 48Q_{0,7}u_{10}u_{40} - 36Q_{1,3}u_{10}u_{41}\\
&- 12Q_{1,3}u_{11}u_{40} - 36Q_{1,3}u_{12} - 72Q_{1,6}u_{20}u_{40} - 576Q_{1,6}u_{21} - 72Q_{1,6}u_{30}^2\\
&+ 12Q_{1,8}u_{40} + 48uu_{01}u_{41} - 48uu_{02}u_{40} + 432uu_{11}u_{31} - 96uu_{12}u_{30}\\
&+ 24uu_{20}^2u_{41} + 48uu_{20}u_{21}u_{40} - 48uu_{20}u_{22} + 96uu_{20}u_{30}u_{31} + 384uu_{21}^2\\
&+ 48uu_{21}u_{30}^2 + 12u_{01}^2u_{40} + 72u_{01}u_{10}u_{30}u_{40} - 144u_{01}u_{10}u_{31}\\
&- 192u_{01}u_{20}u_{21} - 24u_{01}u_{20}u_{30}^2 - 36u_{10}^2u_{20}u_{41} - 36u_{10}^2u_{21}u_{40}\\
&+ 36u_{10}^2u_{22} - 72u_{10}^2u_{30}u_{31} + 576u_{10}u_{11}u_{21} + 72u_{10}u_{11}u_{30}^2 + u_{20}^4u_{40}\\
&+ 32u_{20}^3u_{21} + 4u_{20}^3u_{30}^2))/4\\
D_y\Omega(93) &= 6\Omega(90) + 3\Omega_{yy}u_{30}(2Q_{3,4} - 3Q_{1,3}u_{10} + 4uu_{01} + 2uu_{20}^2 - 3u_{10}^2u_{20})\\
&+ 6\Omega_{xy}u_{21}(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{01} - 2uu_{20}^2 + 3u_{10}^2u_{20})\\
&+ 3\Omega_{xx}u_{12}(2Q_{3,4} - 3Q_{1,3}u_{10} + 4uu_{01} + 2uu_{20}^2 - 3u_{10}^2u_{20})\\
&+ 6\Omega_y(Q_{2,5}u_{21} + 2Q_{3,4}u_{31} - 3Q_{1,3}u_{10}u_{31} + 4uu_{01}u_{31} + 12uu_{11}u_{21}\\
&+ 2uu_{20}^2u_{31} + 4uu_{20}u_{21}u_{30} - 4u_{01}u_{10}u_{21} - 3u_{10}^2u_{20}u_{31} - 3u_{10}^2u_{21}u_{30})\\
&+ 3\Omega_x(-Q_{2,5}u_{12} - 2Q_{3,4}u_{22} + 3Q_{1,3}u_{10}u_{22} - 4uu_{01}u_{22} - 12uu_{11}u_{12}\\
&- 4uu_{12}u_{20}u_{30} - 2uu_{20}^2u_{22} + 4u_{01}u_{10}u_{12} + 3u_{10}^2u_{12}u_{30} + 3u_{10}^2u_{20}u_{22})\\
&+ 3\Omega(2Q_{2,5}u_{22} + 2Q_{3,4}u_{32} - 3Q_{1,3}u_{10}u_{32} + Q_{1,3}u_{12}u_{30} - 18Q_{1,6}u_{12}\\
&+ 4uu_{01}u_{32} + 24uu_{11}u_{22} + 4uu_{12}u_{20}u_{40} + 20uu_{12}u_{21} + 4uu_{12}u_{30}^2\\
&+ 2uu_{20}^2u_{32} + 8uu_{20}u_{22}u_{30} - 8u_{01}u_{10}u_{22} - 6u_{01}u_{12}u_{20} - 3u_{10}^2u_{12}u_{40}\\
&- 3u_{10}^2u_{20}u_{32} - 6u_{10}^2u_{22}u_{30} + 18u_{10}u_{11}u_{12} + u_{12}u_{20}^3)\\
D_x\Omega(94) &= \Omega(91) + \Omega_yu_{10} - \Omega u_{10}u_{40}\\
D_y\Omega(94) &= 2\Omega(86) + \Omega(85)\\
D_x\Omega(82) &= \Omega_{yy}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3\\
&- 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11}\\
&+ 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30}\\
&- 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30}\\
&- 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)\\
&+ \Omega_y(-288Q_{0,12}u_{10}u_{40} + 288Q_{0,12}u_{11} + 216Q_{0,7}^2u_{30}\\
&- 144Q_{0,7}u_{01}u_{20}u_{40} - 144Q_{0,7}u_{01}u_{21} + 288Q_{0,7}u_{01}u_{30}^2 - 144Q_{0,7}u_{02}u_{20}\\
&- 288Q_{0,7}u_{10}u_{11}u_{40} - 288Q_{0,7}u_{10}u_{12} - 288Q_{0,7}u_{10}u_{21}u_{30} - 72Q_{0,7}u_{11}^2\\
&- 432Q_{0,7}u_{11}u_{20}u_{30} - 24Q_{0,7}u_{20}^3u_{40} - 72Q_{0,7}u_{20}^2u_{21} - 72Q_{0,7}u_{20}^2u_{30}^2\\
&+ 72Q_{0,9}u_{01}u_{40} + 72Q_{0,9}u_{02} + 216Q_{0,9}u_{11}u_{30} + 36Q_{0,9}u_{20}^2u_{40}\\
&+ 72Q_{0,9}u_{20}u_{21} + 72Q_{0,9}u_{20}u_{30}^2 + 72Q_{1,11}u_{20}u_{40} + 72Q_{1,11}u_{21}
\end{aligned}
$$

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\begin{aligned}
&+ 72Q_{1,11}u_{30}^2 + Q_{1,13}u_{40} + 144Q_{1,8}u_{11}u_{40} + 144Q_{1,8}u_{12}\\
&+ 144Q_{1,8}u_{21}u_{30} - 195u_{01}^2u_{11}u_{40} - 288u_{01}^2u_{12} - 504u_{01}^2u_{21}u_{30}\\
&+ 156u_{01}u_{02}u_{10}u_{40} - 288u_{01}u_{02}u_{11} - 144u_{01}u_{02}u_{20}u_{30} - 10u_{01}u_{10}^2u_{31}u_{40}\\
&- 20u_{01}u_{10}u_{11}u_{30}u_{40} + 20u_{01}u_{10}u_{20}u_{21}u_{40} - 504u_{01}u_{11}^2u_{30}\\
&+ 82u_{01}u_{11}u_{20}^2u_{40} - 432u_{01}u_{11}u_{20}u_{30}^2 + 144u_{01}u_{12}u_{20}^2 - 96u_{01}u_{20}^3u_{30}u_{40}\\
&- 144u_{01}u_{20}^2u_{30}^3 - 12u_{02}u_{10}^2u_{30}u_{40} + 12u_{02}u_{10}u_{20}^2u_{40} + 72u_{02}u_{11}u_{20}^2\\
&+ 2u_{10}^3u_{22}u_{40} - 6u_{10}^2u_{12}u_{20}u_{40} + 20u_{10}u_{11}^2u_{20}u_{40} + 144u_{11}^3u_{20}\\
&+ 216u_{11}^2u_{20}^2u_{30} + 42u_{11}u_{20}^4u_{40} + 72u_{11}u_{20}^3u_{21} + 72u_{11}u_{20}^3u_{30}^2)\\
&+ \Omega(-288Q_{0,12}u_{10}u_{41} - 288Q_{0,12}u_{11}u_{40} - 864Q_{0,12}u_{12} + 648Q_{0,7}^2u_{31}\\
&- 144Q_{0,7}u_{01}u_{20}u_{41} - 144Q_{0,7}u_{01}u_{21}u_{40} + 144Q_{0,7}u_{01}u_{22}\\
&+ 1008Q_{0,7}u_{01}u_{30}u_{31} + 144Q_{0,7}u_{02}u_{20}u_{40} + 1152Q_{0,7}u_{02}u_{21}\\
&+ 144Q_{0,7}u_{02}u_{30}^2 - 288Q_{0,7}u_{10}u_{11}u_{41} - 864Q_{0,7}u_{10}u_{21}u_{31}\\
&+ 72Q_{0,7}u_{11}^2u_{40} + 432Q_{0,7}u_{11}u_{12} - 1296Q_{0,7}u_{11}u_{20}u_{31}\\
&+ 288Q_{0,7}u_{12}u_{20}u_{30} - 24Q_{0,7}u_{20}^3u_{41} - 72Q_{0,7}u_{20}^2u_{21}u_{40}\\
&+ 72Q_{0,7}u_{20}^2u_{22} - 144Q_{0,7}u_{20}^2u_{30}u_{31} - 1152Q_{0,7}u_{20}u_{21}^2\\
&- 144Q_{0,7}u_{20}u_{21}u_{30}^2 + 72Q_{0,9}u_{01}u_{41} - 72Q_{0,9}u_{02}u_{40} + 648Q_{0,9}u_{11}u_{31}\\
&- 144Q_{0,9}u_{12}u_{30} + 36Q_{0,9}u_{20}^2u_{41} + 72Q_{0,9}u_{20}u_{21}u_{40} - 72Q_{0,9}u_{20}u_{22}\\
&+ 144Q_{0,9}u_{20}u_{30}u_{31} + 576Q_{0,9}u_{21}^2 + 72Q_{0,9}u_{21}u_{30}^2 + 72Q_{1,11}u_{20}u_{41}\\
&+ 72Q_{1,11}u_{21}u_{40} - 72Q_{1,11}u_{22} + 144Q_{1,11}u_{30}u_{31} + Q_{1,13}u_{41}\\
&+ 144Q_{1,8}u_{11}u_{41} + 432Q_{1,8}u_{21}u_{31} - 195u_{01}^2u_{11}u_{41} - 216u_{01}^2u_{21}u_{30}u_{40}\\
&- 864u_{01}^2u_{21}u_{31} + 216u_{01}^2u_{22}u_{30} + 216u_{01}^2u_{30}^2u_{31} + 156u_{01}u_{02}u_{10}u_{41}\\
&+ 288u_{01}u_{02}u_{11}u_{40} + 864u_{01}u_{02}u_{12} + 144u_{01}u_{02}u_{20}u_{30}u_{40}\\
&+ 1152u_{01}u_{02}u_{21}u_{30} + 144u_{01}u_{02}u_{30}^3 - 10u_{01}u_{10}^2u_{31}u_{41}\\
&- 20u_{01}u_{10}u_{11}u_{30}u_{41} + 20u_{01}u_{10}u_{20}u_{21}u_{41} + 72u_{01}u_{11}^2u_{30}u_{40}\\
&- 1296u_{01}u_{11}^2u_{31} + 1008u_{01}u_{11}u_{12}u_{30} + 82u_{01}u_{11}u_{20}^2u_{41}\\
&- 1296u_{01}u_{11}u_{20}u_{30}u_{31} - 2304u_{01}u_{11}u_{21}^2 - 288u_{01}u_{11}u_{21}u_{30}^2\\
&+ 288u_{01}u_{12}u_{20}u_{30}^2 - 96u_{01}u_{20}^3u_{30}u_{41} - 144u_{01}u_{20}^2u_{21}u_{30}u_{40}\\
&+ 432u_{01}u_{20}^2u_{21}u_{31} + 144u_{01}u_{20}^2u_{22}u_{30} - 288u_{01}u_{20}^2u_{30}^2u_{31}\\
&- 1152u_{01}u_{20}u_{21}^2u_{30} - 144u_{01}u_{20}u_{21}u_{30}^3 - 12u_{02}u_{10}^2u_{30}u_{41}\\
&+ 12u_{02}u_{10}u_{20}^2u_{41} - 72u_{02}u_{11}u_{20}^2u_{40} + 2u_{10}^3u_{22}u_{41}\\
&- 6u_{10}^2u_{12}u_{20}u_{41} + 20u_{10}u_{11}^2u_{20}u_{41} - 144u_{11}^3u_{20}u_{40}\\
&- 576u_{11}^3u_{21} - 72u_{11}^3u_{30}^2 - 432u_{11}^2u_{12}u_{20} + 648u_{11}^2u_{20}^2u_{31}\\
&- 144u_{11}u_{12}u_{20}^2u_{30} + 42u_{11}u_{20}^4u_{41} + 72u_{11}u_{20}^3u_{21}u_{40}\\
&- 72u_{11}u_{20}^3u_{22} + 144u_{11}u_{20}^3u_{30}u_{31} + 576u_{11}u_{20}^2u_{21}^2\\
&+ 72u_{11}u_{20}^2u_{21}u_{30}^2)
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(82) = {}& \Omega_{yy}u_{30}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11} - 24Q_{0,7}u_{20}^3\\
&+ 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13} + 144Q_{1,8}u_{11}\\
&- 195u_{01}^2u_{11} + 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31} - 20u_{01}u_{10}u_{11}u_{30}\\
&+ 20u_{01}u_{10}u_{20}u_{21} + 82u_{01}u_{11}u_{20}^2 - 96u_{01}u_{20}^3u_{30} - 12u_{02}u_{10}^2u_{30}\\
&+ 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20} + 20u_{10}u_{11}^2u_{20}\\
&+ 42u_{11}u_{20}^4) + 2\Omega_{xy}u_{21}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11}
\end{aligned}
$$

$$+ 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11}$$
$$+ 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30}$$
$$- 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30}$$
$$- 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)$$
$$+ \Omega_{xx}u_{12}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11} - 24Q_{0,7}u_{20}^3$$
$$+ 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13} + 144Q_{1,8}u_{11} - 195u_{01}^2u_{11}$$
$$+ 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31} - 20u_{01}u_{10}u_{11}u_{30} + 20u_{01}u_{10}u_{20}u_{21}$$
$$+ 82u_{01}u_{11}u_{20}^2 - 96u_{01}u_{20}^3u_{30} - 12u_{02}u_{10}^2u_{30}$$
$$+ 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20} + 20u_{10}u_{11}^2u_{20} + 42u_{11}u_{20}^4)$$
$$+ 2\Omega_y(-288Q_{0,12}u_{10}u_{31} + 216Q_{0,7}^2u_{21}$$
$$- 144Q_{0,7}u_{01}u_{20}u_{31} + 288Q_{0,7}u_{01}u_{21}u_{30}$$
$$- 288Q_{0,7}u_{10}u_{11}u_{31} - 288Q_{0,7}u_{10}u_{21}^2 - 432Q_{0,7}u_{11}u_{20}u_{21} - 24Q_{0,7}u_{20}^3u_{31}$$
$$- 72Q_{0,7}u_{20}^2u_{21}u_{30} + 72Q_{0,9}u_{01}u_{31} + 216Q_{0,9}u_{11}u_{21} + 36Q_{0,9}u_{20}^2u_{31}$$
$$+ 72Q_{0,9}u_{20}u_{21}u_{30} + 72Q_{1,11}u_{20}u_{31} + 72Q_{1,11}u_{21}u_{30} + Q_{1,13}u_{31}$$
$$+ 144Q_{1,8}u_{11}u_{31} + 144Q_{1,8}u_{21}^2 - 195u_{01}^2u_{11}u_{31} - 288u_{01}^2u_{21}^2$$
$$+ 156u_{01}u_{02}u_{10}u_{31} - 10u_{01}u_{10}^2u_{31}^2 - 20u_{01}u_{10}u_{11}u_{30}u_{31}$$
$$+ 20u_{01}u_{10}u_{20}u_{21}u_{31} - 432u_{01}u_{11}^2u_{21} + 82u_{01}u_{11}u_{20}^2u_{31}$$
$$- 432u_{01}u_{11}u_{20}u_{21}u_{30} - 96u_{01}u_{20}^3u_{30}u_{31} + 144u_{01}u_{20}^2u_{21}^2$$
$$- 144u_{01}u_{20}^2u_{21}u_{30}^2 - 12u_{02}u_{10}^2u_{30}u_{31} + 12u_{02}u_{10}u_{20}^2u_{31}$$
$$+ 2u_{10}^3u_{22}u_{31} - 6u_{10}^2u_{12}u_{20}u_{31} + 20u_{10}u_{11}^2u_{20}u_{31} + 216u_{11}^2u_{20}^2u_{21}$$
$$+ 42u_{11}u_{20}^4u_{31} + 72u_{11}u_{20}^3u_{21}u_{30}) + \Omega_x(288Q_{0,12}u_{10}u_{22}$$
$$- 216Q_{0,7}^2u_{12} - 288Q_{0,7}u_{01}u_{12}u_{30}$$
$$+ 144Q_{0,7}u_{01}u_{20}u_{22} + 288Q_{0,7}u_{10}u_{11}u_{22}$$
$$+ 288Q_{0,7}u_{10}u_{12}u_{21} + 432Q_{0,7}u_{11}u_{12}u_{20}$$
$$+ 72Q_{0,7}u_{12}u_{20}^2u_{30} + 24Q_{0,7}u_{20}^3u_{22}$$
$$- 72Q_{0,9}u_{01}u_{22} - 216Q_{0,9}u_{11}u_{12} - 72Q_{0,9}u_{12}u_{20}u_{30} - 36Q_{0,9}u_{20}^2u_{22}$$
$$- 72Q_{1,11}u_{12}u_{30} - 72Q_{1,11}u_{20}u_{22} - Q_{1,13}u_{22} - 144Q_{1,8}u_{11}u_{22}$$
$$+ 195u_{01}^2u_{11}u_{22} + 288u_{01}^2u_{12}u_{21} - 156u_{01}u_{02}u_{10}u_{22} + 10u_{01}u_{10}^2u_{22}u_{31}$$
$$+ 20u_{01}u_{10}u_{11}u_{22}u_{30} - 20u_{01}u_{10}u_{20}u_{21}u_{22} + 432u_{01}u_{11}^2u_{12}$$
$$+ 432u_{01}u_{11}u_{12}u_{20}u_{30} - 82u_{01}u_{11}u_{20}^2u_{22} - 144u_{01}u_{12}u_{20}^2u_{21}$$
$$+ 144u_{01}u_{12}u_{20}^2u_{30}^2 + 96u_{01}u_{20}^3u_{22}u_{30} + 12u_{02}u_{10}^2u_{22}u_{30} - 12u_{02}u_{10}u_{20}^2u_{22}$$
$$- 2u_{10}^3u_{22}^2 + 6u_{10}^2u_{12}u_{20}u_{22} - 20u_{10}u_{11}^2u_{20}u_{22} - 216u_{11}^2u_{12}u_{20}^2$$
$$- 72u_{11}u_{12}u_{20}^3u_{30} - 42u_{11}u_{20}^4u_{22})$$
$$+ \Omega(-288Q_{0,12}u_{10}u_{32} + 288Q_{0,12}u_{12}u_{30}$$
$$+ 432Q_{0,7}^2u_{22} - 144Q_{0,7}u_{01}u_{12}u_{40} - 144Q_{0,7}u_{01}u_{20}u_{32}$$
$$+ 576Q_{0,7}u_{01}u_{22}u_{30} + 432Q_{0,7}u_{02}u_{12}$$
$$- 288Q_{0,7}u_{10}u_{11}u_{32} - 288Q_{0,7}u_{10}u_{12}u_{31}$$
$$- 576Q_{0,7}u_{10}u_{21}u_{22} - 144Q_{0,7}u_{11}u_{12}u_{30}$$
$$- 864Q_{0,7}u_{11}u_{20}u_{22} - 72Q_{0,7}u_{12}u_{20}^2u_{40}$$
$$- 720Q_{0,7}u_{12}u_{20}u_{21} - 144Q_{0,7}u_{12}u_{20}u_{30}^2$$
$$- 24Q_{0,7}u_{20}^3u_{32} - 144Q_{0,7}u_{20}^2u_{22}u_{30}$$

$$
\begin{aligned}
&+ 72Q_{0,9}u_{01}u_{32} + 432Q_{0,9}u_{11}u_{22} + 72Q_{0,9}u_{12}u_{20}u_{40} + 360Q_{0,9}u_{12}u_{21}\\
&+ 72Q_{0,9}u_{12}u_{30}^2 + 36Q_{0,9}u_{20}^2u_{32} + 144Q_{0,9}u_{20}u_{22}u_{30} + 72Q_{1,11}u_{12}u_{40}\\
&+ 72Q_{1,11}u_{20}u_{32} + 144Q_{1,11}u_{22}u_{30} + Q_{1,13}u_{32} + 144Q_{1,8}u_{11}u_{32}\\
&+ 144Q_{1,8}u_{12}u_{31} + 288Q_{1,8}u_{21}u_{22} - 195u_{01}^2u_{11}u_{32} - 216u_{01}^2u_{12}u_{30}u_{40}\\
&- 288u_{01}^2u_{12}u_{31} - 576u_{01}^2u_{21}u_{22} + 156u_{01}u_{02}u_{10}u_{32} + 144u_{01}u_{02}u_{12}u_{30}\\
&- 10u_{01}u_{10}^2u_{31}u_{32} - 20u_{01}u_{10}u_{11}u_{30}u_{32} + 20u_{01}u_{10}u_{20}u_{21}u_{32}\\
&- 864u_{01}u_{11}^2u_{22} - 1440u_{01}u_{11}u_{12}u_{21} - 432u_{01}u_{11}u_{12}u_{30}^2\\
&+ 82u_{01}u_{11}u_{20}^2u_{32} - 864u_{01}u_{11}u_{20}u_{22}u_{30} - 144u_{01}u_{12}u_{20}^2u_{30}u_{40}\\
&+ 144u_{01}u_{12}u_{20}^2u_{31} - 720u_{01}u_{12}u_{20}u_{21}u_{30} - 144u_{01}u_{12}u_{20}u_{30}^3\\
&- 96u_{01}u_{20}^3u_{30}u_{32} + 288u_{01}u_{20}^2u_{21}u_{22} - 288u_{01}u_{20}^2u_{22}u_{30}^2\\
&- 12u_{02}u_{10}^2u_{30}u_{32} + 12u_{02}u_{10}u_{20}^2u_{32}\\
&+ 2u_{10}^3u_{22}u_{32} - 6u_{10}^2u_{12}u_{20}u_{32} + 20u_{10}u_{11}^2u_{20}u_{32}\\
&- 216u_{11}^3u_{12} + 144u_{11}^2u_{12}u_{20}u_{30}\\
&+ 432u_{11}^2u_{20}^2u_{22} + 72u_{11}u_{12}u_{20}^3u_{40} + 360u_{11}u_{12}u_{20}^2u_{21} + 72u_{11}u_{12}u_{20}^2u_{30}^2\\
&+ 42u_{11}u_{20}^4u_{32} + 144u_{11}u_{20}^3u_{22}u_{30}).
\end{aligned}
$$

3.5. ℓ^*-covering.

3.5.1. *Concervation laws and nonlocalities.*

$$
\begin{aligned}
D_x\Pi(76) &= \Pi_{yy}\\
D_y\Pi(76) &= -\Pi_{yy}u_{30} + 2\Pi_{xy}u_{21} - \Pi_{xx}u_{12}\\
D_x\Pi(77) &= -\Pi_{yy}u_{10} + \Pi_y(u_{11} + u_{20}u_{30}) + \Pi(-u_{12} + u_{20}u_{31} + u_{21}u_{30})\\
D_y\Pi(77) &= \Pi_{yy}u_{10}u_{30} - 2\Pi_{xy}u_{10}u_{21} + \Pi_{xx}u_{10}u_{12} + 2\Pi_yu_{20}u_{21}\\
&\quad - \Pi_xu_{12}u_{20} + \Pi(u_{12}u_{30} + u_{20}u_{22})\\
D_x\Pi(78) &= -\Pi_{yy}u_{01} + \Pi_y(u_{02} + u_{11}u_{30}) + \Pi(u_{11}u_{31} + u_{21}^2)\\
D_y\Pi(78) &= \Pi_{yy}u_{01}u_{30} - 2\Pi_{xy}u_{01}u_{21} + \Pi_{xx}u_{01}u_{12} + 2\Pi_yu_{11}u_{21}\\
&\quad - \Pi_xu_{11}u_{12} + \Pi(u_{11}u_{22} + u_{12}u_{21})\\
D_x\Pi(79) &= (\Pi_{yy}(-Q_{2,5} - 8u_{01}u_{10})\\
&\quad + \Pi_y(-3Q_{0,9} - 18Q_{1,6}u_{30} + 6u_{01}u_{11} + 6u_{01}u_{20}u_{30}\\
&\quad + 12u_{02}u_{10} + 12u_{10}u_{11}u_{30} + u_{20}^3u_{30})\\
&\quad + \Pi(-36Q_{0,7}u_{21} - 18Q_{1,6}u_{31} - 6u_{01}u_{12} + 6u_{01}u_{20}u_{31} - 30u_{01}u_{21}u_{30}\\
&\quad - 12u_{02}u_{11} - 6u_{02}u_{20}u_{30} + 12u_{10}u_{11}u_{31} + 12u_{10}u_{21}^2 - 3u_{11}^2u_{30}\\
&\quad + 36u_{11}u_{20}u_{21} - 3u_{12}u_{20}^2 + u_{20}^3u_{31} + 3u_{20}^2u_{21}u_{30}))/3\\
D_y\Pi(79) &= (\Pi_{yy}u_{30}(Q_{2,5} + 8u_{01}u_{10}) + 2\Pi_{xy}u_{21}(-Q_{2,5} - 8u_{01}u_{10})\\
&\quad + \Pi_{xx}u_{12}(Q_{2,5} + 8u_{01}u_{10})\\
&\quad + 2\Pi_yu_{21}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + \Pi_xu_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&\quad + \Pi(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22}\\
&\quad + 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22}))/3\\
D_x\Pi(80) &= \Pi_{yy}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&\quad + \Pi_y(-4Q_{0,12} + 2Q_{0,7}u_{11} + 2Q_{0,7}u_{20}u_{30} - Q_{0,9}u_{30} + 4u_{01}u_{02}\\
&\quad + 6u_{01}u_{11}u_{30} + 2u_{01}u_{20}u_{30}^2 - 2u_{11}^2u_{20} - u_{11}u_{20}^2u_{30})\\
&\quad + \Pi(-2Q_{0,7}u_{12} + 2Q_{0,7}u_{20}u_{31} + 2Q_{0,7}u_{21}u_{30} - Q_{0,9}u_{31}
\end{aligned}
$$

$$
\begin{aligned}
&+ 4u_{01}u_{11}u_{31} - 2u_{01}u_{12}u_{30} + 2u_{01}u_{20}u_{30}u_{31} + 4u_{01}u_{21}^2\\
&+ 2u_{01}u_{21}u_{30}^2 - 2u_{02}^2 - 2u_{02}u_{11}u_{30} + 6u_{11}^2u_{21} - u_{11}u_{20}^2u_{31})\\
D_y\Pi(80) &= \Pi_{yy}u_{30}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 2\Pi_{xy}u_{21}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ \Pi_{xx}u_{12}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 2\Pi_y u_{21}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ \Pi_x u_{12}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ \Pi(2Q_{0,7}u_{12}u_{30} + 2Q_{0,7}u_{20}u_{22} - Q_{0,9}u_{22} + 4u_{01}u_{11}u_{22}\\
&+ 4u_{01}u_{12}u_{21} + 2u_{01}u_{12}u_{30}^2 + 2u_{01}u_{20}u_{22}u_{30} + 3u_{11}^2u_{12} - u_{11}u_{20}^2u_{22})\\
D_x\Pi(81) &= (2\Pi_{yy}(120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6}\\
&- 180Q_{1,8}u_{10} - 570u_{01}^2u_{10} + 2070u_{01}u_{10}u_{20}^2)\\
&+ \Pi_y(540Q_{2,10}u_{30} - 240Q_{2,5}u_{02} - 240Q_{2,5}u_{11}u_{30} + 1440Q_{0,12}u_{10}\\
&- 1620Q_{0,7}Q_{1,3}u_{30} - 2160Q_{0,7}Q_{1,6} - 2160Q_{0,7}u_{10}u_{11}\\
&- 2160Q_{0,7}u_{10}u_{20}u_{30} - 360Q_{0,9}u_{10}u_{30} + 5Q_{1,13} + 2160Q_{1,6}u_{11}u_{20}\\
&+ 1080Q_{1,6}u_{20}^2u_{30} + 360Q_{1,8}u_{11} + 360Q_{1,8}u_{20}u_{30} + 105u_{01}^2u_{11}\\
&- 2520u_{01}^2u_{20}u_{30} - 1140u_{01}u_{02}u_{10} - 50u_{01}u_{10}^2u_{31} - 220u_{01}u_{10}u_{11}u_{30}\\
&+ 100u_{01}u_{10}u_{20}u_{21} + 8280u_{01}u_{10}u_{20}u_{30}^2 - 670u_{01}u_{11}u_{20}^2\\
&+ 5640u_{01}u_{20}^3u_{30} - 1680u_{02}u_{10}^2u_{30} + 60u_{02}u_{10}u_{20}^2 + 10u_{10}^3u_{22}\\
&- 30u_{10}^2u_{12}u_{20} + 1540u_{10}u_{11}^2u_{20} - 4140u_{10}u_{11}u_{20}^2u_{30} - 6u_{20}^5u_{30})\\
&+ 6\Pi(90Q_{2,10}u_{31} - 40Q_{2,5}u_{11}u_{31} - 40Q_{2,5}u_{21}^2 - 480Q_{0,12}u_{11}\\
&- 240Q_{0,12}u_{20}u_{30} + 180Q_{0,7}^2u_{30} - 270Q_{0,7}Q_{1,3}u_{31} + 360Q_{0,7}u_{01}u_{30}^2\\
&+ 360Q_{0,7}u_{10}u_{12} - 360Q_{0,7}u_{10}u_{20}u_{31} - 360Q_{0,7}u_{10}u_{21}u_{30}\\
&+ 360Q_{0,7}u_{11}^2 - 120Q_{0,9}u_{02} - 60Q_{0,9}u_{10}u_{31} - 60Q_{0,9}u_{11}u_{30}\\
&- 360Q_{1,11}u_{21} + 360Q_{1,6}u_{01}u_{31} - 360Q_{1,6}u_{02}u_{30} + 2160Q_{1,6}u_{11}u_{21}\\
&- 360Q_{1,6}u_{12}u_{20} + 180Q_{1,6}u_{20}^2u_{31} + 360Q_{1,6}u_{20}u_{21}u_{30} - 60Q_{1,8}u_{12}\\
&+ 60Q_{1,8}u_{20}u_{31} + 60Q_{1,8}u_{21}u_{30} + 60u_{01}^2u_{12} - 600u_{01}^2u_{20}u_{31}\\
&+ 300u_{01}^2u_{21}u_{30} + 180u_{01}^2u_{30}^3 + 240u_{01}u_{02}u_{11} + 120u_{01}u_{02}u_{20}u_{30}\\
&+ 220u_{01}u_{10}u_{11}u_{31} + 240u_{01}u_{10}u_{12}u_{30} + 1380u_{01}u_{10}u_{20}u_{30}u_{31}\\
&- 320u_{01}u_{10}u_{21}^2 - 240u_{01}u_{10}u_{21}u_{30}^2 + 420u_{01}u_{11}^2u_{30} - 720u_{01}u_{11}u_{20}u_{21}\\
&+ 940u_{01}u_{20}^3u_{31} + 360u_{01}u_{20}^2u_{21}u_{30} + 240u_{02}^2u_{10} - 270u_{02}u_{10}^2u_{31}\\
&+ 240u_{02}u_{10}u_{11}u_{30} + 20u_{02}u_{20}^3u_{30} - 720u_{10}u_{11}^2u_{21} - 690u_{10}u_{11}u_{20}^2u_{31}\\
&- 360u_{11}^3u_{20} - 150u_{11}^2u_{20}^2u_{30} - 120u_{11}u_{20}^3u_{21} + 5u_{12}u_{20}^4 - u_{20}^5u_{31}\\
&- 5u_{20}^4u_{21}u_{30}))/2\\
D_y\Pi(81) &= \Pi_{yy}u_{30}(-120Q_{2,5}u_{01} - Q_{3,9} - 540Q_{0,7}u_{10}^2 - 540Q_{1,3}Q_{1,6}\\
&+ 180Q_{1,8}u_{10} + 570u_{01}^2u_{10} - 2070u_{01}u_{10}u_{20}^2)\\
&+ 2\Pi_{xy}u_{21}(120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6}\\
&- 180Q_{1,8}u_{10} - 570u_{01}^2u_{10} + 2070u_{01}u_{10}u_{20}^2)\\
&+ \Pi_{xx}u_{12}(-120Q_{2,5}u_{01} - Q_{3,9} - 540Q_{0,7}u_{10}^2 - 540Q_{1,3}Q_{1,6}\\
&+ 180Q_{1,8}u_{10} + 570u_{01}^2u_{10} - 2070u_{01}u_{10}u_{20}^2)\\
&+ 6\Pi_y u_{21}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20}\\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2 + 60Q_{1,8}u_{20} - 600u_{01}^2u_{20}
\end{aligned}
$$

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\begin{aligned}
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2\\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 3\Pi_x u_{12}(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3}\\
&+ 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20}\\
&+ 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3\\
&+ 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5)\\
&+ 3\Pi(90Q_{2,10}u_{22} - 40Q_{2,5}u_{11}u_{22} - 40Q_{2,5}u_{12}u_{21} - 270Q_{0,7}Q_{1,3}u_{22}\\
&- 360Q_{0,7}u_{10}u_{12}u_{30} - 360Q_{0,7}u_{10}u_{20}u_{22} - 60Q_{0,9}u_{10}u_{22}\\
&- 180Q_{1,11}u_{12} + 360Q_{1,6}u_{01}u_{22} + 1080Q_{1,6}u_{11}u_{12} + 360Q_{1,6}u_{12}u_{20}u_{30}\\
&+ 180Q_{1,6}u_{20}^2u_{22} + 60Q_{1,8}u_{12}u_{30} + 60Q_{1,8}u_{20}u_{22} + 120u_{01}^2u_{12}u_{30}\\
&- 600u_{01}^2u_{20}u_{22} + 220u_{01}u_{10}u_{11}u_{22} - 320u_{01}u_{10}u_{12}u_{21}\\
&- 240u_{01}u_{10}u_{12}u_{30}^2 + 1380u_{01}u_{10}u_{20}u_{22}u_{30} - 360u_{01}u_{11}u_{12}u_{20}\\
&+ 180u_{01}u_{12}u_{20}^2u_{30} + 940u_{01}u_{20}^3u_{22} - 270u_{02}u_{10}^2u_{22} - 360u_{10}u_{11}^2u_{12}\\
&- 690u_{10}u_{11}u_{20}^2u_{22} - 60u_{11}u_{12}u_{20}^3 - 5u_{12}u_{20}^4u_{30} - u_{20}^5u_{22})\\
D_x\Pi(82) &= \Pi_{yy}(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} + 65u_{01}^3\\
&+ 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21} + 566u_{01}u_{10}u_{11}u_{20}\\
&+ 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4\\
&- 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40} + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31}\\
&- 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{21}\\
&- 48u_{10}u_{11}u_{20}^3) + \Pi_y(-288Q_{0,12}u_{10}u_{30} + 72Q_{0,7}Q_{0,9} - 144Q_{0,7}u_{01}u_{11}\\
&- 144Q_{0,7}u_{01}u_{20}u_{30} - 288Q_{0,7}u_{02}u_{10} - 288Q_{0,7}u_{10}u_{11}u_{30}\\
&- 24Q_{0,7}u_{20}^3u_{30} + 144Q_{0,9}u_{01}u_{30} + 72Q_{0,9}u_{11}u_{20} + 36Q_{0,9}u_{20}^2u_{30}\\
&+ 72Q_{1,11}u_{11} + 72Q_{1,11}u_{20}u_{30} + Q_{1,13}u_{30} + 8Q_{1,16} + 144Q_{1,8}u_{02}\\
&+ 144Q_{1,8}u_{11}u_{30} - 288u_{01}^2u_{02} - 411u_{01}^2u_{11}u_{30} - 4u_{01}u_{02}u_{10}u_{30}\\
&+ 224u_{01}u_{02}u_{20}^2 + 40u_{01}u_{10}^2u_{22} - 10u_{01}u_{10}^2u_{30}u_{31} - 20u_{01}u_{10}u_{11}u_{30}^2\\
&- 80u_{01}u_{10}u_{12}u_{20} + 20u_{01}u_{10}u_{20}u_{21}u_{30} + 288u_{01}u_{11}^2u_{20}\\
&+ 10u_{01}u_{11}u_{20}^2u_{30} - 96u_{01}u_{20}^3u_{30}^2 - 80u_{02}u_{10}^2u_{21} - 12u_{02}u_{10}^2u_{30}^2\\
&+ 160u_{02}u_{10}u_{11}u_{20} + 12u_{02}u_{10}u_{20}^2u_{30} + 2u_{10}^3u_{22}u_{30}\\
&- 40u_{10}^2u_{11}u_{12} - 6u_{10}^2u_{12}u_{20}u_{30} + 20u_{10}u_{11}^2u_{20}u_{30} + 24u_{11}^2u_{20}^3\\
&+ 42u_{11}u_{20}^4u_{30}) + \Pi(-576Q_{0,12}u_{02} - 288Q_{0,12}u_{10}u_{31} - 288Q_{0,12}u_{11}u_{30}\\
&+ 432Q_{0,7}^2u_{21} + 144Q_{0,7}u_{01}u_{12} - 144Q_{0,7}u_{01}u_{20}u_{31} + 720Q_{0,7}u_{01}u_{21}u_{30}\\
&+ 288Q_{0,7}u_{02}u_{11} + 144Q_{0,7}u_{02}u_{20}u_{30} - 288Q_{0,7}u_{10}u_{11}u_{31}\\
&- 288Q_{0,7}u_{10}u_{21}^2 + 72Q_{0,7}u_{11}^2u_{30} - 864Q_{0,7}u_{11}u_{20}u_{21}\\
&+ 72Q_{0,7}u_{12}u_{20}^2 - 24Q_{0,7}u_{20}^3u_{31} - 72Q_{0,7}u_{20}^2u_{21}u_{30} + 72Q_{0,9}u_{01}u_{31}\\
&- 72Q_{0,9}u_{02}u_{30} + 432Q_{0,9}u_{11}u_{21} - 72Q_{0,9}u_{12}u_{20} + 36Q_{0,9}u_{20}^2u_{31}\\
&+ 72Q_{0,9}u_{20}u_{21}u_{30} - 72Q_{1,11}u_{12} + 72Q_{1,11}u_{20}u_{31} + 72Q_{1,11}u_{21}u_{30}\\
&+ Q_{1,13}u_{31} + 144Q_{1,8}u_{11}u_{31} + 144Q_{1,8}u_{21}^2 - 195u_{01}^2u_{11}u_{31}\\
&+ 216u_{01}^2u_{12}u_{30} - 288u_{01}^2u_{21}^2 + 216u_{01}^2u_{21}u_{30}^2 + 288u_{01}u_{02}^2\\
&+ 156u_{01}u_{02}u_{10}u_{31} + 576u_{01}u_{02}u_{11}u_{30} + 144u_{01}u_{02}u_{20}u_{30}^2\\
&- 10u_{01}u_{10}^2u_{31}^2 - 20u_{01}u_{10}u_{11}u_{30}u_{31} + 20u_{01}u_{10}u_{20}u_{21}u_{31}\\
&- 864u_{01}u_{11}^2u_{21} + 72u_{01}u_{11}^2u_{30}^2\\
&+ 82u_{01}u_{11}u_{20}^2u_{31} - 864u_{01}u_{11}u_{20}u_{21}u_{30} + 144u_{01}u_{12}u_{20}^2u_{30}
\end{aligned}
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\begin{aligned}
&- 96u_{01}u_{20}^3u_{30}u_{31} + 144u_{01}u_{20}^2u_{21}^2 - 144u_{01}u_{20}^2u_{21}u_{30}^2\\
&- 12u_{02}u_{10}^2u_{30}u_{31} + 12u_{02}u_{10}u_{20}^2u_{31} - 288u_{02}u_{11}^2u_{20}\\
&- 72u_{02}u_{11}u_{20}^2u_{30} + 2u_{10}^3u_{22}u_{31} - 6u_{10}^2u_{12}u_{20}u_{31}\\
&+ 20u_{10}u_{11}^2u_{20}u_{31} - 36u_{11}^4 - 144u_{11}^3u_{20}u_{30} + 432u_{11}^2u_{20}^2u_{21}\\
&- 72u_{11}u_{12}u_{20}^3 + 42u_{11}u_{20}^4u_{31} + 72u_{11}u_{20}^3u_{21}u_{30})\\
D_y\Pi(82) &= \Pi_{yy}u_{30}(Q_{2,12} - 288Q_{0,7}u_{01}u_{10} + 72Q_{1,11}u_{10} + 144Q_{1,8}u_{01}\\
&- 65u_{01}^3 - 72u_{01}^2u_{10}u_{30} + 324u_{01}^2u_{20}^2 - 514u_{01}u_{10}^2u_{21}\\
&- 566u_{01}u_{10}u_{11}u_{20} - 72u_{01}u_{10}u_{20}^2u_{30} - 6u_{01}u_{20}^4 + 42u_{02}u_{10}^2u_{20}\\
&- 2u_{10}^4u_{41} + 12u_{10}^3u_{11}u_{40} - 16u_{10}^3u_{12} + 8u_{10}^3u_{20}u_{31} + 28u_{10}^3u_{21}u_{30}\\
&+ 257u_{10}^2u_{11}^2 - 36u_{10}^2u_{11}u_{20}u_{30} - 24u_{10}^2u_{20}^2u_{21} + 48u_{10}u_{11}u_{20}^3)\\
&+ 2\Pi_{xy}u_{21}(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01}\\
&+ 65u_{01}^3 + 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21}\\
&+ 566u_{01}u_{10}u_{11}u_{20} + 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4 - 42u_{02}u_{10}^2u_{20}\\
&+ 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40} + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31}\\
&- 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{21}\\
&- 48u_{10}u_{11}u_{20}^3) + \Pi_{xx}u_{12}(Q_{2,12} - 288Q_{0,7}u_{01}u_{10} + 72Q_{1,11}u_{10}\\
&+ 144Q_{1,8}u_{01} - 65u_{01}^3 - 72u_{01}^2u_{10}u_{30}\\
&+ 324u_{01}^2u_{20}^2 - 514u_{01}u_{10}^2u_{21} - 566u_{01}u_{10}u_{11}u_{20} - 72u_{01}u_{10}u_{20}^2u_{30}\\
&- 6u_{01}u_{20}^4 + 42u_{02}u_{10}^2u_{20} - 2u_{10}^4u_{41} + 12u_{10}^3u_{11}u_{40}\\
&- 16u_{10}^3u_{12} + 8u_{10}^3u_{20}u_{31} + 28u_{10}^3u_{21}u_{30} + 257u_{10}^2u_{11}^2\\
&- 36u_{10}^2u_{11}u_{20}u_{30} - 24u_{10}^2u_{20}^2u_{21} + 48u_{10}u_{11}u_{20}^3)\\
&+ 2\Pi_yu_{21}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11}\\
&- 24Q_{0,7}u_{20}^3 + 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13}\\
&+ 144Q_{1,8}u_{11} - 195u_{01}^2u_{11} + 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31}\\
&- 20u_{01}u_{10}u_{11}u_{30} + 20u_{01}u_{10}u_{20}u_{21} + 82u_{01}u_{11}u_{20}^2\\
&- 96u_{01}u_{20}^3u_{30} - 12u_{02}u_{10}^2u_{30}\\
&+ 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20} + 20u_{10}u_{11}^2u_{20} + 42u_{11}u_{20}^4)\\
&+ \Pi_xu_{12}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3\\
&- 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11} + 195u_{01}^2u_{11}\\
&- 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30} - 20u_{01}u_{10}u_{20}u_{21}\\
&- 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30}\\
&- 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)\\
&+ \Pi(-288Q_{0,12}u_{10}u_{22} + 216Q_{0,7}^2u_{12} + 288Q_{0,7}u_{01}u_{12}u_{30}\\
&- 144Q_{0,7}u_{01}u_{20}u_{22} - 288Q_{0,7}u_{10}u_{11}u_{22}\\
&- 288Q_{0,7}u_{10}u_{12}u_{21} - 432Q_{0,7}u_{11}u_{12}u_{20} - 72Q_{0,7}u_{12}u_{20}^2u_{30}\\
&- 24Q_{0,7}u_{20}^3u_{22} + 72Q_{0,9}u_{01}u_{22} + 216Q_{0,9}u_{11}u_{12} + 72Q_{0,9}u_{12}u_{20}u_{30}\\
&+ 36Q_{0,9}u_{20}^2u_{22} + 72Q_{1,11}u_{12}u_{30} + 72Q_{1,11}u_{20}u_{22}\\
&+ Q_{1,13}u_{22} + 144Q_{1,8}u_{11}u_{22} + 144Q_{1,8}u_{12}u_{21} - 195u_{01}^2u_{11}u_{22}\\
&- 288u_{01}^2u_{12}u_{21} + 156u_{01}u_{02}u_{10}u_{22}\\
&- 10u_{01}u_{10}^2u_{22}u_{31} - 20u_{01}u_{10}u_{11}u_{22}u_{30} + 20u_{01}u_{10}u_{20}u_{21}u_{22}
\end{aligned}
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\begin{aligned}
&- 432u_{01}u_{11}^2u_{12} - 432u_{01}u_{11}u_{12}u_{20}u_{30} + 82u_{01}u_{11}u_{20}^2u_{22}\\
&+ 144u_{01}u_{12}u_{20}^2u_{21} - 144u_{01}u_{12}u_{20}^2u_{30}^2\\
&- 96u_{01}u_{20}^3u_{22}u_{30} - 12u_{02}u_{10}^2u_{22}u_{30} + 12u_{02}u_{10}u_{20}^2u_{22} + 2u_{10}^3u_{22}^2\\
&- 6u_{10}^2u_{12}u_{20}u_{22} + 20u_{10}u_{11}^2u_{20}u_{22} + 216u_{11}^2u_{12}u_{20}^2\\
&+ 72u_{11}u_{12}u_{20}^3u_{30} + 42u_{11}u_{20}^4u_{22})\\
D_x\Pi(83) &= \Pi(76) + \Pi_y u_{30} + \Pi u_{31}\\
D_y\Pi(83) &= 2\Pi_y u_{21} - \Pi_x u_{12} + \Pi u_{22}\\
D_x\Pi(84) &= \Pi(77) - 4\Pi_{yy}u + \Pi_y(4u_{01} + 3u_{10}u_{30})\\
&+ \Pi(-4u_{02} + 3u_{10}u_{31} - 3u_{11}u_{30} + 4u_{20}u_{21})\\
D_y\Pi(84) &= 4\Pi_{yy}uu_{30} - 8\Pi_{xy}uu_{21} + 4\Pi_{xx}uu_{12} + 6\Pi_y u_{10}u_{21} - 3\Pi_x u_{10}u_{12}\\
&+ \Pi(3u_{10}u_{22} + 2u_{12}u_{20})\\
D_x\Pi(85) &= (4\Pi(78) - \Pi_{yy}Q_{1,3} + \Pi_y(2Q_{0,7} - 2u_{11}u_{20} - u_{20}^2u_{30})\\
&+ \Pi(-2u_{01}u_{31} + 2u_{02}u_{30} - 12u_{11}u_{21} + 2u_{12}u_{20} - u_{20}^2u_{31}\\
&- 2u_{20}u_{21}u_{30}))/4\\
D_y\Pi(85) &= (\Pi_{yy}Q_{1,3}u_{30} - 2\Pi_{xy}Q_{1,3}u_{21} + \Pi_{xx}Q_{1,3}u_{12} + 2\Pi_y u_{21}(-2u_{01} - u_{20}^2)\\
&+ \Pi_x u_{12}(2u_{01} + u_{20}^2)\\
&+ \Pi(-2u_{01}u_{22} - 6u_{11}u_{12} - 2u_{12}u_{20}u_{30} - u_{20}^2u_{22}))/4\\
D_x\Pi(86) &= (3\Pi(79) + 3\Pi_{yy}(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01})\\
&+ \Pi_y(7Q_{2,5}u_{30} + 30Q_{0,7}u_{10} + 15Q_{1,3}u_{11} + 15Q_{1,3}u_{20}u_{30} - 24Q_{1,8}\\
&+ 48uu_{02} + 48uu_{11}u_{30} - 28u_{01}u_{10}u_{30} - 24u_{01}u_{20}^2\\
&+ 18u_{10}u_{11}u_{20} + 9u_{10}u_{20}^2u_{30})\\
&+ \Pi(7Q_{2,5}u_{31} + 96Q_{0,12} - 60Q_{0,7}u_{11} - 30Q_{0,7}u_{20}u_{30} + 21Q_{0,9}u_{30}\\
&- 15Q_{1,3}u_{12} + 15Q_{1,3}u_{20}u_{31} + 15Q_{1,3}u_{21}u_{30} - 216Q_{1,6}u_{21} + 48uu_{11}u_{31}\\
&+ 48uu_{21}^2 - 48u_{01}u_{02} - 10u_{01}u_{10}u_{31} - 84u_{01}u_{11}u_{30}\\
&- 30u_{01}u_{20}u_{30}^2 - 18u_{02}u_{10}u_{30} + 108u_{10}u_{11}u_{21} - 18u_{10}u_{12}u_{20}\\
&+ 9u_{10}u_{20}^2u_{31} + 18u_{10}u_{20}u_{21}u_{30} + 60u_{11}^2u_{20} + 21u_{11}u_{20}^2u_{30}))/3\\
D_y\Pi(86) &= (3\Pi_{yy}u_{30}(8Q_{3,4} - 3Q_{1,3}u_{10} + 16uu_{01})\\
&+ 6\Pi_{xy}u_{21}(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01})\\
&+ 3\Pi_{xx}u_{12}(8Q_{3,4} - 3Q_{1,3}u_{10} + 16uu_{01})\\
&+ 2\Pi_y u_{21}(7Q_{2,5} + 15Q_{1,3}u_{20} + 48uu_{11} - 10u_{01}u_{10} + 9u_{10}u_{20}^2)\\
&+ \Pi_x u_{12}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 48uu_{11} + 10u_{01}u_{10} - 9u_{10}u_{20}^2)\\
&+ \Pi(7Q_{2,5}u_{22} + 15Q_{1,3}u_{12}u_{30} + 15Q_{1,3}u_{20}u_{22} - 108Q_{1,6}u_{12}\\
&+ 48uu_{11}u_{22} + 48uu_{12}u_{21}\\
&- 10u_{01}u_{10}u_{22} + 54u_{10}u_{11}u_{12} + 18u_{10}u_{12}u_{20}u_{30} + 9u_{10}u_{20}^2u_{22}))/3\\
D_x\Pi(87) &= (120\Pi(80) + 2\Pi_{yy}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 5\Pi_y(-24Q_{0,7}u_{10}u_{30} - 12Q_{1,11} + 24Q_{1,3}u_{02} + 24Q_{1,3}u_{11}u_{30}\\
&+ 72Q_{1,6}u_{11} + 72Q_{1,6}u_{20}u_{30} + 12Q_{1,8}u_{30} - 24u_{01}u_{11}u_{20} + 12u_{01}u_{20}^2u_{30}\\
&- 4u_{11}u_{20}^3 - u_{20}^4u_{30}) + 5\Pi(-48Q_{0,12}u_{30} - 48Q_{0,7}u_{02} - 24Q_{0,7}u_{10}u_{31}\\
&- 24Q_{0,7}u_{11}u_{30} + 72Q_{0,9}u_{21} + 24Q_{1,3}u_{11}u_{31} + 24Q_{1,3}u_{21}^2 - 72Q_{1,6}u_{12}\\
&+ 72Q_{1,6}u_{20}u_{31} + 72Q_{1,6}u_{21}u_{30} + 12Q_{1,8}u_{31} - 12u_{01}^2u_{31} - 24u_{01}u_{02}u_{30}\\
&- 144u_{01}u_{11}u_{21} - 24u_{01}u_{11}u_{30}^2 + 24u_{01}u_{12}u_{20} - 24u_{01}u_{20}u_{21}u_{30}\\
&+ 48u_{02}u_{11}u_{20} + 12u_{02}u_{20}^2u_{30} - 24u_{11}^3 - 12u_{11}^2u_{20}u_{30} + 4u_{12}u_{20}^3
\end{aligned}
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\begin{aligned}
&- u_{20}^4u_{31} - 4u_{20}^3u_{21}u_{30}))/120\\
D_y\Pi(87) &= (2\Pi_{yy}u_{30}(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)\\
&+ 4\Pi_{xy}u_{21}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 2\Pi_{xx}u_{12}(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)\\
&+ 10\Pi_yu_{21}(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8}\\
&- 12u_{01}^2 - u_{20}^4) + 5\Pi_xu_{12}(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8}\\
&+ 12u_{01}^2 + u_{20}^4) + 5\Pi(-24Q_{0,7}u_{10}u_{22} + 36Q_{0,9}u_{12} + 24Q_{1,3}u_{11}u_{22}\\
&+ 24Q_{1,3}u_{12}u_{21} + 72Q_{1,6}u_{12}u_{30} + 72Q_{1,6}u_{20}u_{22} + 12Q_{1,8}u_{22} - 12u_{01}^2u_{22}\\
&- 72u_{01}u_{11}u_{12} - 24u_{01}u_{12}u_{20}u_{30} - 4u_{12}u_{20}^3u_{30} - u_{20}^4u_{22}))/120\\
D_x\Pi(88) &= \Pi_y\\
D_y\Pi(88) &= \Pi(76)\\
D_x\Pi(89) &= \Pi_{yy}u + \Pi_y(-2u_{01} - u_{10}u_{30}) + \Pi(3u_{02} - u_{10}u_{31} + 2u_{11}u_{30} - 2u_{20}u_{21})\\
D_y\Pi(89) &= \Pi(78) - \Pi_{yy}uu_{30} + 2\Pi_{xy}uu_{21} - \Pi_{xx}uu_{12} - 2\Pi_yu_{10}u_{21}\\
&+ \Pi_xu_{10}u_{12} - \Pi(u_{10}u_{22} + u_{12}u_{20})\\
D_x\Pi(90) &= \Pi_{yy}(3Q_{3,4} - Q_{1,3}u_{10} + 4uu_{01}) + \Pi_y(-Q_{2,5}u_{30} - 6Q_{0,7}u_{10}\\
&- 2Q_{1,3}u_{11} - 2Q_{1,3}u_{20}u_{30} + 4Q_{1,8} - 4uu_{02} - 4uu_{11}u_{30} + 4u_{01}u_{10}u_{30}\\
&+ 4u_{01}u_{20}^2 - 2u_{10}u_{11}u_{20} - u_{10}u_{20}^2u_{30}) + \Pi(-Q_{2,5}u_{31} - 20Q_{0,12}\\
&+ 12Q_{0,7}u_{11} + 6Q_{0,7}u_{20}u_{30} - 4Q_{0,9}u_{30} + 2Q_{1,3}u_{12} - 2Q_{1,3}u_{20}u_{31}\\
&- 2Q_{1,3}u_{21}u_{30} + 36Q_{1,6}u_{21} - 4uu_{11}u_{31} - 4uu_{21}^2 + 8u_{01}u_{02} + 2u_{01}u_{10}u_{31}\\
&+ 16u_{01}u_{11}u_{30} + 6u_{01}u_{20}u_{30}^2 + 2u_{02}u_{10}u_{30} - 12u_{10}u_{11}u_{21} + 2u_{10}u_{12}u_{20}\\
&- u_{10}u_{20}^2u_{31} - 2u_{10}u_{20}u_{21}u_{30} - 12u_{11}^2u_{20} - 4u_{11}u_{20}^2u_{30})\\
D_y\Pi(90) &= \Pi(80) + \Pi_{yy}u_{30}(-3Q_{3,4} + Q_{1,3}u_{10} - 4uu_{01})\\
&+ 2\Pi_{xy}u_{21}(3Q_{3,4} - Q_{1,3}u_{10} + 4uu_{01})\\
&+ \Pi_{xx}u_{12}(-3Q_{3,4} + Q_{1,3}u_{10} - 4uu_{01}) + 2\Pi_yu_{21}(-Q_{2,5} - 2Q_{1,3}u_{20}\\
&- 4uu_{11} + 2u_{01}u_{10} - u_{10}u_{20}^2) + \Pi_xu_{12}(Q_{2,5} + 2Q_{1,3}u_{20} + 4uu_{11}\\
&- 2u_{01}u_{10} + u_{10}u_{20}^2) + \Pi(-Q_{2,5}u_{22} - 2Q_{1,3}u_{12}u_{30} - 2Q_{1,3}u_{20}u_{22}\\
&+ 18Q_{1,6}u_{12} - 4uu_{11}u_{22} - 4uu_{12}u_{21} + 2u_{01}u_{10}u_{22} - 6u_{10}u_{11}u_{12}\\
&- 2u_{10}u_{12}u_{20}u_{30} - u_{10}u_{20}^2u_{22})\\
D_x\Pi(91) &= \Pi\\
D_y\Pi(91) &= \Pi(88)\\
D_x\Pi(92) &= \Pi(88) + \Pi u_{30}\\
D_y\Pi(92) &= \Pi(83)\\
D_x\Pi(93) &= \Pi(83) - 2\Pi u_{21}\\
D_y\Pi(93) &= -\Pi u_{12}\\
D_x\Pi(94) &= (4\Pi(89) + \Pi(88)(2u_{01} - u_{20}^2) + 2\Pi(84) + 4\Pi(83)u_{10} + 4\Pi(76)u\\
&+ \Pi(4Q_{0,7} + 6u_{01}u_{30} - 12u_{10}u_{21} - 4u_{11}u_{20} - u_{20}^2u_{30}))/4\\
D_y\Pi(94) &= (2\Pi(88)(Q_{0,7} + u_{01}u_{30} - u_{11}u_{20}) + 4\Pi(85) + 4\Pi(83)u_{01}\\
&+ \Pi(76)Q_{1,3} - 6\Pi u_{10}u_{12})/4\\
D_x\Pi(95) &= (4\Pi(85) + \Pi(83)(2u_{01} - u_{20}^2) + 2\Pi(77)u_{20}\\
&+ \Pi(76)(Q_{1,3} + 2u_{10}u_{20}) + 2\Pi(Q_{0,7}u_{30} + u_{01}u_{30}^2 + u_{11}^2))/4\\
D_y\Pi(95) &= (\Pi(83)(Q_{0,7} + u_{01}u_{30} - u_{11}u_{20}) + \Pi(77)u_{11} + \Pi(76)u_{10}u_{11})/2\\
D_x\Pi(96) &= (24\Pi(90) + 24\Pi(89)(-2u_{01} + u_{20}^2) + \Pi(88)(24Q_{0,7}u_{10} + 72Q_{1,6}u_{20}
\end{aligned}
$$

$$
\begin{aligned}
&- 12Q_{1,8} - 12u_{01}^2 - u_{20}^4) + 12\Pi(86) - 96\Pi(85)u_{10} \\
&+ 12\Pi(84)(-2u_{01} + u_{20}^2) + 8\Pi(83)(Q_{2,5} - 10u_{01}u_{10} + 3u_{10}u_{20}^2) \\
&- 96\Pi(78)u + 12\Pi(77)(-Q_{1,3} - 6u_{10}u_{20}) \\
&+ 24\Pi(76)(Q_{3,4} - 2Q_{1,3}u_{10} - 2uu_{01} + uu_{20}^2 - 3u_{10}^2u_{20}) \\
&+ \Pi(-24Q_{2,5}u_{21} - 96Q_{0,7}u_{01} - 48Q_{0,7}u_{10}u_{30} - 24Q_{1,11} + 144Q_{1,6}u_{11} \\
&+ 72Q_{1,6}u_{20}u_{30} - 12Q_{1,8}u_{30} - 84u_{01}^2u_{30} + 96u_{01}u_{10}u_{21} \\
&- 72u_{01}u_{10}u_{30}^2 + 48u_{01}u_{11}u_{20} + 24u_{01}u_{20}^2u_{30} - 72u_{10}u_{11}^2 \\
&- 8u_{11}u_{20}^3 - u_{20}^4u_{30}))/24 \\
D_y\Pi(96) &= (120\Pi(89)(-Q_{0,7} - u_{01}u_{30} + u_{11}u_{20}) + 10\Pi(88)(-12Q_{0,7}u_{01} \\
&- 3Q_{1,11} + 18Q_{1,6}u_{11} - 9u_{01}^2u_{30} + 6u_{01}u_{11}u_{20} + 3u_{01}u_{20}^2u_{30} - u_{11}u_{20}^3) \\
&+ 60\Pi(87) - 240\Pi(85)u_{01} + 60\Pi(84)(-Q_{0,7} - u_{01}u_{30} + u_{11}u_{20}) \\
&+ 60\Pi(83)(-Q_{1,8} - 2u_{01}^2 - 2u_{01}u_{10}u_{30} - u_{01}u_{20}^2 + 2u_{10}u_{11}u_{20}) \\
&- 60\Pi(78)Q_{1,3} - 180\Pi(77)u_{10}u_{11} + \Pi(76)(3Q_{2,7} - 120Q_{0,7}u - 60Q_{1,3}u_{01} \\
&+ 180Q_{1,6}u_{10} - 120uu_{01}u_{30} + 120uu_{11}u_{20} - 180u_{10}^2u_{11} - u_{10}u_{20}^3) \\
&+ 30\Pi u_{12}(-Q_{2,5} + 4u_{01}u_{10}))/60 \\
D_x\Pi(97) &= 3\Pi(93) + \Pi(88)u_{20} + \Pi(2u_{11} + u_{20}u_{30}) \\
D_y\Pi(97) &= \Pi(88)u_{11} + \Pi(77) + \Pi(76)u_{10} \\
D_x\Pi(98) &= (36\Pi(95) + \Pi(88)(18Q_{1,6} + 6u_{01}u_{20} - u_{20}^3) \\
&+ 3\Pi(83)(-3Q_{1,3} - 2u_{10}u_{20}) - 6\Pi(77)u_{10} + 12\Pi(76)uu_{20} + 12\Pi_{yy}uu_{10} \\
&- 6\Pi_y u_{10}^2u_{30} + \Pi(6Q_{0,9} + 18Q_{1,3}u_{21} + 18Q_{1,6}u_{30} + 12u_{01}u_{11} \\
&+ 6u_{01}u_{20}u_{30} - 12u_{02}u_{10} - 6u_{10}^2u_{31} - 6u_{10}u_{11}u_{30} - u_{20}^3u_{30}))/36 \\
D_y\Pi(98) &= (3\Pi(88)(Q_{0,9} + 2u_{01}u_{11}) - 6\Pi(83)u_{10}u_{11} - 3\Pi(79) + 12\Pi(77)u_{01} \\
&+ \Pi(76)(-Q_{2,5} + 12uu_{11} + 16u_{01}u_{10}) - 12\Pi_{yy}uu_{10}u_{30} \\
&+ 24\Pi_{xy}uu_{10}u_{21} - 12\Pi_{xx}uu_{10}u_{12} - 12\Pi_y u_{10}^2u_{21} + 6\Pi_x u_{10}^2u_{12} \\
&+ 3\Pi(3Q_{1,3}u_{12} - 2u_{10}^2u_{22}))/36 \\
D_x\Pi(99) &= \Pi(97) + \Pi(88)u_{10} + \Pi(2u_{01} + u_{10}u_{30}) \\
D_y\Pi(99) &= 2\Pi(89) + 2\Pi(88)u_{01} + \Pi(84) + 2\Pi(76)u \\
D_x\Pi(100) &= (36\Pi(98) + 12\Pi(97)(2u_{01} - u_{20}^2) - 24\Pi(94)u_{20} \\
&+ 6\Pi(93)(6Q_{1,3} - 2uu_{30} + u_{10}u_{20}) + \Pi(92)(-9Q_{1,3}u_{30} - 18Q_{1,6} \\
&+ 12uu_{10}u_{50} + 24uu_{20}u_{40} + 12uu_{30}^2 - 18u_{01}u_{20} + 24u_{10}^2u_{40} + 6u_{10}u_{11} \\
&+ 30u_{10}u_{20}u_{30} + 7u_{20}^3) + 6\Pi(91)(4Q_{0,7}u_{20} - Q_{0,9} - 3Q_{1,3}u_{21} \\
&- 2uu_{10}u_{30}u_{50} - 2uu_{10}u_{40}^2 - 2uu_{10}u_{41} - 6uu_{20}u_{30}u_{40} - 6uu_{20}u_{31} \\
&- 4uu_{21}u_{30} - 2uu_{30}^3 - 10u_{01}u_{11} + 4u_{01}u_{20}u_{30} + 2u_{02}u_{10} - 6u_{10}^2u_{30}u_{40} \\
&- 8u_{10}^2u_{31} - 18u_{10}u_{20}u_{21} - 6u_{10}u_{20}u_{30}^2))/36 \\
D_y\Pi(100) &= (24\Pi(97)(Q_{0,7} + u_{01}u_{30} - u_{11}u_{20}) - 24\Pi(94)u_{11} \\
&+ 6\Pi(93)(-2uu_{21} - 2u_{01}u_{20} + 3u_{10}u_{11}) + 3\Pi(92)(-6Q_{0,7}u_{20} - Q_{0,9} \\
&- 3Q_{1,3}u_{21} + 4uu_{10}u_{41} + 4uu_{11}u_{40} + 4uu_{20}u_{31} + 4uu_{21}u_{30} \\
&+ 4u_{01}u_{10}u_{40} - 2u_{01}u_{20}u_{30} + 2u_{02}u_{10} + 4u_{10}^2u_{31} + 8u_{10}u_{11}u_{30} \\
&- 2u_{10}u_{20}u_{21} + 5u_{11}u_{20}^2) + 6\Pi(91)(-4Q_{0,12} - 2Q_{0,7}u_{11} - 3Q_{1,3}u_{12} \\
&- 2uu_{10}u_{30}u_{41} - 2uu_{10}u_{31}u_{40} - 2uu_{10}u_{32} - 2uu_{11}u_{30}u_{40} \\
&- 2uu_{11}u_{31} - 4uu_{20}u_{22} - 4uu_{20}u_{30}u_{31} - 4uu_{21}^2 - 2uu_{21}u_{30}^2 - 4u_{01}u_{02} \\
&- 2u_{01}u_{10}u_{30}u_{40} - 2u_{01}u_{10}u_{31} - 2u_{01}u_{11}u_{30} - 4u_{01}u_{20}u_{21}
\end{aligned}
$$

$$
\begin{aligned}
&- 2u_{01}u_{20}u_{30}^2 - 6u_{10}^2u_{22} - 4u_{10}^2u_{30}u_{31} - 12u_{10}u_{11}u_{21} - 4u_{10}u_{11}u_{30}^2\\
&- 2u_{10}u_{12}u_{20} + 2u_{11}^2u_{20}) - 6\Pi(90) + 24\Pi(89)u_{01} + 3\Pi(88)(2Q_{0,7}u_{10}\\
&- 3Q_{1,3}u_{11} - 4uu_{10}u_{30}u_{40} - 8uu_{10}u_{31} - 4uu_{11}u_{30} - 8uu_{20}u_{21}\\
&- 4uu_{20}u_{30}^2 - 2u_{01}u_{10}u_{30} - 12u_{10}^2u_{21} - 4u_{10}^2u_{30}^2 - 2u_{10}u_{11}u_{20})\\
&- 3\Pi(86) - 24\Pi(85)u_{10}\\
&+ 12\Pi(84)u_{01} + \Pi(83)(Q_{2,5} - 9Q_{1,3}u_{20} + 12uu_{10}u_{40} + 12uu_{11}\\
&+ 12uu_{20}u_{30} - 10u_{01}u_{10} + 12u_{10}^2u_{30} - 3u_{10}u_{20}^2) + 24\Pi(78)u\\
&+ 12\Pi(77)Q_{1,3} + 3\Pi(76)(-2Q_{3,4} + 3Q_{1,3}u_{10} + 8uu_{01} - 4uu_{10}u_{30})\\
&+ 24\Pi_y uu_{10}u_{21} - 12\Pi_x uu_{10}u_{12} + 6\Pi(2uu_{10}u_{22}\\
&+ 2uu_{12}u_{20} + 3u_{10}^2u_{12}))/36
\end{aligned}
$$

3.6. **Transformation of vector-valued results.**

$$
\begin{aligned}
\Pi(76) &= \Pi(101)\\
\Pi(77) &= \Pi(102)\\
\Pi(78) &= \Pi(103)\\
\Pi(79) &= \Pi(104)\\
\Pi(80) &= \Pi(105)\\
\Pi(81) &= \Pi(106)\\
\Pi(82) &= \Pi(107)\\
\Pi(83) &= \Pi(108) + \Pi(101)x\\
\Pi(84) &= \Pi(109) + \Pi(102)x\\
\Pi(85) &= \Pi(110) + \Pi(103)x\\
\Pi(86) &= \Pi(111) + \Pi(104)x\\
\Pi(87) &= \Pi(112) + \Pi(105)x\\
\Pi(88) &= \Pi(113) + \Pi(101)y\\
\Pi(89) &= \Pi(114) + \Pi(103)y\\
\Pi(90) &= \Pi(115) + \Pi(105)y\\
\Pi(91) &= (2\Pi(116) + 2\Pi(113)y + \Pi(101)y^2)/2\\
\Pi(92) &= \Pi(117) + \Pi(113)x + \Pi(108)y + \Pi(101)xy\\
\Pi(93) &= (2\Pi(118) + 2\Pi(108)x + \Pi(101)x^2)/2\\
\Pi(94) &= (4\Pi(119) + 4\Pi(114)x + \Pi(113)Q_{1,3} + 4\Pi(110)y + 2\Pi(109)x\\
&\quad + 4\Pi(108)u + 4\Pi(103)xy + \Pi(102)x^2 + 4\Pi(101)xu + \Pi(101)yQ_{1,3})/4\\
\Pi(95) &= (4\Pi(120) + 4\Pi(110)x + \Pi(108)Q_{1,3} + 2\Pi(103)x^2\\
&\quad + 2\Pi(102)u_{10} + \Pi(101)xQ_{1,3})/4\\
\Pi(96) &= (60\Pi(121) + 60\Pi(115)x - 60\Pi(114)Q_{1,3} + \Pi(113)(3Q_{2,7} + 180Q_{1,6}u_{10}\\
&\quad - u_{10}u_{20}^3) + 60\Pi(112)y + 30\Pi(111)x - 240\Pi(110)u - 30\Pi(109)Q_{1,3}\\
&\quad + 60\Pi(108)(Q_{3,4} - Q_{1,3}u_{10}) + 60\Pi(105)xy + 15\Pi(104)x^2 - 240\Pi(103)xu\\
&\quad - 60\Pi(103)yQ_{1,3} - 30\Pi(102)xQ_{1,3} - 90\Pi(102)u_{10}^2 + 60\Pi(101)x(Q_{3,4}\\
&\quad - Q_{1,3}u_{10}) + \Pi(101)y(3Q_{2,7} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)\\
&\quad - 60\Pi(101)(Q_{1,3}u + u_{10}^3))/60\\
\Pi(97) &= (2\Pi(122) + 6\Pi(118)x + 2\Pi(113)u_{10} + 3\Pi(108)x^2\\
&\quad + 2\Pi(102)y + \Pi(101)x^3 + 2\Pi(101)yu_{10})/2\\
\Pi(98) &= (36\Pi(123) + 36\Pi(120)x + \Pi(113)(-Q_{2,5} + 4u_{01}u_{10}) + 18\Pi(110)x^2
\end{aligned}
$$

$$
\begin{aligned}
&- 3\Pi(108)u_{10}^2 - 3\Pi(104)y + 6\Pi(103)x^3 + 12\Pi(102)u - 12\Pi(101)xu_{10}^2 \\
&+ \Pi(101)y(-Q_{2,5} + 4u_{01}u_{10}) + 12\Pi(101)uu_{10})/36 \\
\Pi(99) &= (8\Pi(124) + 8\Pi(122)x + 12\Pi(118)x^2 + 16\Pi(114)y + 16\Pi(113)u \\
&+ 8\Pi(109)y + 4\Pi(108)x^3 + 8\Pi(103)y^2 + 8\Pi(102)xy + \Pi(101)x^4 \\
&+ 16\Pi(101)yu)/8 \\
\Pi(100) &= (72\Pi(125) + 72\Pi(123)x + 24\Pi(122)Q_{1,3} + 36\Pi(120)x^2 - 48\Pi(119)u_{10} \\
&+ 72\Pi(118)xQ_{1,3} + 6\Pi(118)(-4uu_{20} + 3u_{10}^2) \\
&+ 2\Pi(117)(Q_{2,5} - 9Q_{1,3}u_{20} + 12uu_{10}u_{40} + 12uu_{20}u_{30} + 2u_{01}u_{10} \\
&+ 12u_{10}^2u_{30} - 3u_{10}u_{20}^2) + 12\Pi(116)(4Q_{0,7}u_{10} - 3Q_{1,3}u_{11} - Q_{1,8} \\
&- 2uu_{10}u_{30}u_{40} - 2uu_{10}u_{31} - 4uu_{20}u_{21} - 2uu_{20}u_{30}^2 - 2u_{01}^2 + 2u_{01}u_{10}u_{30} \\
&- u_{01}u_{20}^2 - 6u_{10}^2u_{21} - 2u_{10}^2u_{30}^2 - 2u_{10}u_{11}u_{20}) - 12\Pi(115)y \\
&- 48\Pi(114)xu_{10} + 48\Pi(114)u + 2\Pi(113)x(Q_{2,5} - 9Q_{1,3}u_{20} + 12uu_{10}u_{40} \\
&+ 12uu_{20}u_{30} + 2u_{01}u_{10} + 12u_{10}^2u_{30} - 3u_{10}u_{20}^2) + 12\Pi(113)y(4Q_{0,7}u_{10} \\
&- 3Q_{1,3}u_{11} - Q_{1,8} - 2uu_{10}u_{30}u_{40} - 2uu_{10}u_{31} - 4uu_{20}u_{21} - 2uu_{20}u_{30}^2 \\
&- 2u_{01}^2 + 2u_{01}u_{10}u_{30} - u_{01}u_{20}^2 - 6u_{10}^2u_{21} - 2u_{10}^2u_{30}^2 - 2u_{10}u_{11}u_{20}) \\
&+ 6\Pi(113)(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{10}u_{30}) - 6\Pi(111)y \\
&+ 12\Pi(110)x^3 - 48\Pi(110)yu_{10} - 24\Pi(109)xu_{10} + 24\Pi(109)u \\
&+ 36\Pi(108)x^2Q_{1,3} + 6\Pi(108)x(-4uu_{20} + 3u_{10}^2) \\
&+ 2\Pi(108)y(Q_{2,5} - 9Q_{1,3}u_{20} + 12uu_{10}u_{40} + 12uu_{20}u_{30} + 2u_{01}u_{10} \\
&+ 12u_{10}^2u_{30} - 3u_{10}u_{20}^2) - 24\Pi(108)uu_{10} - 6\Pi(105)y^2 - 6\Pi(104)xy \\
&+ 3\Pi(103)x^4 - 48\Pi(103)xyu_{10} + 48\Pi(103)yu \\
&- 12\Pi(102)x^2u_{10} + 24\Pi(102)xu + 24\Pi(102)yQ_{1,3} + 12\Pi(101)x^3Q_{1,3} \\
&+ 2\Pi(101)xy(Q_{2,5} - 9Q_{1,3}u_{20} + 12uu_{10}u_{40} + 12uu_{20}u_{30} + 2u_{01}u_{10} \\
&+ 12u_{10}^2u_{30} - 3u_{10}u_{20}^2) - 24\Pi(101)xuu_{10} \\
&+ 6\Pi(101)y^2(4Q_{0,7}u_{10} - 3Q_{1,3}u_{11} - Q_{1,8} - 2uu_{10}u_{30}u_{40} - 2uu_{10}u_{31} \\
&- 4uu_{20}u_{21} - 2uu_{20}u_{30}^2 - 2u_{01}^2 + 2u_{01}u_{10}u_{30} - u_{01}u_{20}^2 - 6u_{10}^2u_{21} \\
&- 2u_{10}^2u_{30}^2 - 2u_{10}u_{11}u_{20}) \\
&+ 6\Pi(101)y(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{10}u_{30}) + 24\Pi(101)u^2)/72
\end{aligned}
$$

3.7. **Transformation of results.** According to the transformations of nonlocal *vector-* and *form-valued* variables, the associated x and y derivatives are *explicitly* given by

$$
\begin{aligned}
D_x\Pi(101) &= \Pi_{yy} \\
D_y\Pi(101) &= -\Pi_{yy}u_{30} + 2\Pi_{xy}u_{21} - \Pi_{xx}u_{12} \\
D_x\Pi(102) &= -\Pi_{yy}u_{10} + \Pi_y(u_{11} + u_{20}u_{30}) + \Pi(-u_{12} + u_{20}u_{31} + u_{21}u_{30}) \\
D_y\Pi(102) &= \Pi_{yy}u_{10}u_{30} - 2\Pi_{xy}u_{10}u_{21} + \Pi_{xx}u_{10}u_{12} + 2\Pi_y u_{20}u_{21} - \Pi_x u_{12}u_{20} \\
&+ \Pi(u_{12}u_{30} + u_{20}u_{22}) \\
D_x\Pi(103) &= -\Pi_{yy}u_{01} + \Pi_y(u_{02} + u_{11}u_{30}) + \Pi(u_{11}u_{31} + u_{21}^2) \\
D_y\Pi(103) &= \Pi_{yy}u_{01}u_{30} - 2\Pi_{xy}u_{01}u_{21} + \Pi_{xx}u_{01}u_{12} + 2\Pi_y u_{11}u_{21} - \Pi_x u_{11}u_{12} \\
&+ \Pi(u_{11}u_{22} + u_{12}u_{21}) \\
D_x\Pi(104) &= (\Pi_{yy}(-Q_{2,5} - 8u_{01}u_{10}) + \Pi_y(-3Q_{0,9} - 18Q_{1,6}u_{30} + 6u_{01}u_{11} \\
&+ 6u_{01}u_{20}u_{30} + 12u_{02}u_{10} + 12u_{10}u_{11}u_{30} + u_{20}^3u_{30}) \\
&+ \Pi(-36Q_{0,7}u_{21} - 18Q_{1,6}u_{31} - 6u_{01}u_{12} + 6u_{01}u_{20}u_{31} - 30u_{01}u_{21}u_{30} \\
&- 12u_{02}u_{11} - 6u_{02}u_{20}u_{30} + 12u_{10}u_{11}u_{31} + 12u_{10}u_{21}^2
\end{aligned}
$$

$$
\begin{aligned}
&\quad - 3u_{11}^2u_{30} + 36u_{11}u_{20}u_{21} - 3u_{12}u_{20}^2 + u_{20}^3u_{31} + 3u_{20}^2u_{21}u_{30}))/3 \\
D_y\Pi(104) &= (\Pi_{yy}u_{30}(Q_{2,5} + 8u_{01}u_{10}) + 2\Pi_{xy}u_{21}(-Q_{2,5} - 8u_{01}u_{10}) \\
&\quad + \Pi_{xx}u_{12}(Q_{2,5} + 8u_{01}u_{10}) + 2\Pi_y u_{21}(-18Q_{1,6} + 6u_{01}u_{20} \\
&\quad + 12u_{10}u_{11} + u_{20}^3) + \Pi_x u_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3) \\
&\quad + \Pi(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22} \\
&\quad + 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} \\
&\quad + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22}))/3 \\
D_x\Pi(105) &= \Pi_{yy}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) \\
&\quad + \Pi_y(-4Q_{0,12} + 2Q_{0,7}u_{11} + 2Q_{0,7}u_{20}u_{30} - Q_{0,9}u_{30} + 4u_{01}u_{02} \\
&\quad + 6u_{01}u_{11}u_{30} + 2u_{01}u_{20}u_{30}^2 - 2u_{11}^2u_{20} - u_{11}u_{20}^2u_{30}) \\
&\quad + \Pi(-2Q_{0,7}u_{12} + 2Q_{0,7}u_{20}u_{31} + 2Q_{0,7}u_{21}u_{30} - Q_{0,9}u_{31} + 4u_{01}u_{11}u_{31} \\
&\quad - 2u_{01}u_{12}u_{30} + 2u_{01}u_{20}u_{30}u_{31} + 4u_{01}u_{21}^2 + 2u_{01}u_{21}u_{30}^2 \\
&\quad - 2u_{02}^2 - 2u_{02}u_{11}u_{30} + 6u_{11}^2u_{21} - u_{11}u_{20}^2u_{31}) \\
D_y\Pi(105) &= \Pi_{yy}u_{30}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2) \\
&\quad + 2\Pi_{xy}u_{21}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) \\
&\quad + \Pi_{xx}u_{12}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2) \\
&\quad + 2\Pi_y u_{21}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2) \\
&\quad + \Pi_x u_{12}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2) \\
&\quad + \Pi(2Q_{0,7}u_{12}u_{30} + 2Q_{0,7}u_{20}u_{22} - Q_{0,9}u_{22} + 4u_{01}u_{11}u_{22} + 4u_{01}u_{12}u_{21} \\
&\quad + 2u_{01}u_{12}u_{30}^2 + 2u_{01}u_{20}u_{22}u_{30} + 3u_{11}^2u_{12} - u_{11}u_{20}^2u_{22}) \\
D_x\Pi(106) &= (2\Pi_{yy}(120Q_{2,5}u_{01} + Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6} - 180Q_{1,8}u_{10} \\
&\quad - 570u_{01}^2u_{10} + 2070u_{01}u_{10}u_{20}^2) + \Pi_y(540Q_{2,10}u_{30} - 240Q_{2,5}u_{02} \\
&\quad - 240Q_{2,5}u_{11}u_{30} + 1440Q_{0,12}u_{10} - 1620Q_{0,7}Q_{1,3}u_{30} - 2160Q_{0,7}Q_{1,6} \\
&\quad - 2160Q_{0,7}u_{10}u_{11} - 2160Q_{0,7}u_{10}u_{20}u_{30} - 360Q_{0,9}u_{10}u_{30} + 5Q_{1,13} \\
&\quad + 2160Q_{1,6}u_{11}u_{20} + 1080Q_{1,6}u_{20}^2u_{30} + 360Q_{1,8}u_{11} + 360Q_{1,8}u_{20}u_{30} \\
&\quad + 105u_{01}^2u_{11} - 2520u_{01}^2u_{20}u_{30} - 1140u_{01}u_{02}u_{10} - 50u_{01}u_{10}^2u_{31} \\
&\quad - 220u_{01}u_{10}u_{11}u_{30} + 100u_{01}u_{10}u_{20}u_{21} + 8280u_{01}u_{10}u_{20}u_{30}^2 \\
&\quad - 670u_{01}u_{11}u_{20}^2 + 5640u_{01}u_{20}^3u_{30} - 1680u_{02}u_{10}^2u_{30} \\
&\quad + 60u_{02}u_{10}u_{20}^2 + 10u_{10}^3u_{22} - 30u_{10}^2u_{12}u_{20} + 1540u_{10}u_{11}^2u_{20} \\
&\quad - 4140u_{10}u_{11}u_{20}^2u_{30} - 6u_{20}^5u_{30}) + 6\Pi(90Q_{2,10}u_{31} - 40Q_{2,5}u_{11}u_{31} \\
&\quad - 40Q_{2,5}u_{21}^2 - 480Q_{0,12}u_{11} - 240Q_{0,12}u_{20}u_{30} + 180Q_{0,7}^2u_{30} \\
&\quad - 270Q_{0,7}Q_{1,3}u_{31} + 360Q_{0,7}u_{01}u_{30}^2 + 360Q_{0,7}u_{10}u_{12} \\
&\quad - 360Q_{0,7}u_{10}u_{20}u_{31} - 360Q_{0,7}u_{10}u_{21}u_{30} + 360Q_{0,7}u_{11}^2 - 120Q_{0,9}u_{02} \\
&\quad - 60Q_{0,9}u_{10}u_{31} - 60Q_{0,9}u_{11}u_{30} - 360Q_{1,11}u_{21} + 360Q_{1,6}u_{01}u_{31} \\
&\quad - 360Q_{1,6}u_{02}u_{30} + 2160Q_{1,6}u_{11}u_{21} - 360Q_{1,6}u_{12}u_{20} \\
&\quad + 180Q_{1,6}u_{20}^2u_{31} + 360Q_{1,6}u_{20}u_{21}u_{30} - 60Q_{1,8}u_{12} + 60Q_{1,8}u_{20}u_{31} \\
&\quad + 60Q_{1,8}u_{21}u_{30} + 60u_{01}^2u_{12} - 600u_{01}^2u_{20}u_{31} + 300u_{01}^2u_{21}u_{30} \\
&\quad + 180u_{01}^2u_{30}^3 + 240u_{01}u_{02}u_{11} + 120u_{01}u_{02}u_{20}u_{30} + 220u_{01}u_{10}u_{11}u_{31} \\
&\quad + 240u_{01}u_{10}u_{12}u_{30} + 1380u_{01}u_{10}u_{20}u_{30}u_{31} - 320u_{01}u_{10}u_{21}^2 \\
&\quad - 240u_{01}u_{10}u_{21}u_{30}^2 + 420u_{01}u_{11}^2u_{30} - 720u_{01}u_{11}u_{20}u_{21} + 940u_{01}u_{20}^3u_{31} \\
&\quad + 360u_{01}u_{20}^2u_{21}u_{30} + 240u_{02}^2u_{10} - 270u_{02}u_{10}^2u_{31} + 240u_{02}u_{10}u_{11}u_{30}
\end{aligned}
$$

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\begin{aligned}
D_y\Pi(106) &= \Pi_{yy}u_{30}(-120Q_{2,5}u_{01} - Q_{3,9} - 540Q_{0,7}u_{10}^2 - 540Q_{1,3}Q_{1,6} \\
&+ 180Q_{1,8}u_{10} + 570u_{01}^2u_{10} - 2070u_{01}u_{10}u_{20}^2) + 2\Pi_{xy}u_{21}(120Q_{2,5}u_{01} \\
&+ Q_{3,9} + 540Q_{0,7}u_{10}^2 + 540Q_{1,3}Q_{1,6} - 180Q_{1,8}u_{10} - 570u_{01}^2u_{10} \\
&+ 2070u_{01}u_{10}u_{20}^2) + \Pi_{xx}u_{12}(-120Q_{2,5}u_{01} - Q_{3,9} - 540Q_{0,7}u_{10}^2 \\
&- 540Q_{1,3}Q_{1,6} + 180Q_{1,8}u_{10} + 570u_{01}^2u_{10} - 2070u_{01}u_{10}u_{20}^2) \\
&+ 6\Pi_y u_{21}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20} \\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2 + 60Q_{1,8}u_{20} - 600u_{01}^2u_{20} \\
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2 \\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 3\Pi_x u_{12}(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} \\
&+ 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} \\
&+ 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 \\
&+ 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5) + 3\Pi(90Q_{2,10}u_{22} - 40Q_{2,5}u_{11}u_{22} \\
&- 40Q_{2,5}u_{12}u_{21} - 270Q_{0,7}Q_{1,3}u_{22} - 360Q_{0,7}u_{10}u_{12}u_{30} \\
&- 360Q_{0,7}u_{10}u_{20}u_{22} - 60Q_{0,9}u_{10}u_{22} - 180Q_{1,11}u_{12} + 360Q_{1,6}u_{01}u_{22} \\
&+ 1080Q_{1,6}u_{11}u_{12} + 360Q_{1,6}u_{12}u_{20}u_{30} + 180Q_{1,6}u_{20}^2u_{22} \\
&+ 60Q_{1,8}u_{12}u_{30} + 60Q_{1,8}u_{20}u_{22} + 120u_{01}^2u_{12}u_{30} - 600u_{01}^2u_{20}u_{22} \\
&+ 220u_{01}u_{10}u_{11}u_{22} - 320u_{01}u_{10}u_{12}u_{21} - 240u_{01}u_{10}u_{12}u_{30}^2 \\
&+ 1380u_{01}u_{10}u_{20}u_{22}u_{30} - 360u_{01}u_{11}u_{12}u_{20} + 180u_{01}u_{12}u_{20}^2u_{30} \\
&+ 940u_{01}u_{20}^3u_{22} - 270u_{02}u_{10}^2u_{22} - 360u_{10}u_{11}^2u_{12} - 690u_{10}u_{11}u_{20}^2u_{22} \\
&- 60u_{11}u_{12}u_{20}^3 - 5u_{12}u_{20}^4u_{30} - u_{20}^5u_{22}) \\
D_x\Pi(107) &= \Pi_{yy}(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01} \\
&+ 65u_{01}^3 + 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21} + 566u_{01}u_{10}u_{11}u_{20} \\
&+ 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4 - 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} \\
&- 12u_{10}^3u_{11}u_{40} + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31} - 28u_{10}^3u_{21}u_{30} \\
&- 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{21} - 48u_{10}u_{11}u_{20}^3) \\
&+ \Pi_y(-288Q_{0,12}u_{10}u_{30} + 72Q_{0,7}Q_{0,9} - 144Q_{0,7}u_{01}u_{11} \\
&- 144Q_{0,7}u_{01}u_{20}u_{30} - 288Q_{0,7}u_{02}u_{10} - 288Q_{0,7}u_{10}u_{11}u_{30} \\
&- 24Q_{0,7}u_{20}^3u_{30} + 144Q_{0,9}u_{01}u_{30} + 72Q_{0,9}u_{11}u_{20} + 36Q_{0,9}u_{20}^2u_{30} \\
&+ 72Q_{1,11}u_{11} + 72Q_{1,11}u_{20}u_{30} + Q_{1,13}u_{30} + 8Q_{1,16} + 144Q_{1,8}u_{02} \\
&+ 144Q_{1,8}u_{11}u_{30} - 288u_{01}^2u_{02} - 411u_{01}^2u_{11}u_{30} - 4u_{01}u_{02}u_{10}u_{30} \\
&+ 224u_{01}u_{02}u_{20}^2 + 40u_{01}u_{10}^2u_{22} - 10u_{01}u_{10}^2u_{30}u_{31} \\
&- 20u_{01}u_{10}u_{11}u_{30}^2 - 80u_{01}u_{10}u_{12}u_{20} + 20u_{01}u_{10}u_{20}u_{21}u_{30} \\
&+ 288u_{01}u_{11}^2u_{20} + 10u_{01}u_{11}u_{20}^2u_{30} - 96u_{01}u_{20}^3u_{30}^2 - 80u_{02}u_{10}^2u_{21} \\
&- 12u_{02}u_{10}^2u_{30}^2 + 160u_{02}u_{10}u_{11}u_{20} + 12u_{02}u_{10}u_{20}^2u_{30} + 2u_{10}^3u_{22}u_{30} \\
&- 40u_{10}^2u_{11}u_{12} - 6u_{10}^2u_{12}u_{20}u_{30} + 20u_{10}u_{11}^2u_{20}u_{30} + 24u_{11}^2u_{20}^3 \\
&+ 42u_{11}u_{20}^4u_{30}) + \Pi(-576Q_{0,12}u_{02} - 288Q_{0,12}u_{10}u_{31} \\
&- 288Q_{0,12}u_{11}u_{30} + 432Q_{0,7}^2u_{21} + 144Q_{0,7}u_{01}u_{12} - 144Q_{0,7}u_{01}u_{20}u_{31} \\
&+ 720Q_{0,7}u_{01}u_{21}u_{30} + 288Q_{0,7}u_{02}u_{11} + 144Q_{0,7}u_{02}u_{20}u_{30} \\
&- 288Q_{0,7}u_{10}u_{11}u_{31} - 288Q_{0,7}u_{10}u_{21}^2 + 72Q_{0,7}u_{11}^2u_{30} \\
&- 864Q_{0,7}u_{11}u_{20}u_{21} \\
&+ 72Q_{0,7}u_{12}u_{20}^2 - 24Q_{0,7}u_{20}^3u_{31} - 72Q_{0,7}u_{20}^2u_{21}u_{30} + 72Q_{0,9}u_{01}u_{31}
\end{aligned}
$$

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\begin{aligned}
&- 72Q_{0,9}u_{02}u_{30} + 432Q_{0,9}u_{11}u_{21} - 72Q_{0,9}u_{12}u_{20} + 36Q_{0,9}u_{20}^2u_{31}\\
&+ 72Q_{0,9}u_{20}u_{21}u_{30} - 72Q_{1,11}u_{12} + 72Q_{1,11}u_{20}u_{31} + 72Q_{1,11}u_{21}u_{30}\\
&+ Q_{1,13}u_{31} + 144Q_{1,8}u_{11}u_{31} + 144Q_{1,8}u_{21}^2 - 195u_{01}^2u_{11}u_{31}\\
&+ 216u_{01}^2u_{12}u_{30} - 288u_{01}^2u_{21}^2 + 216u_{01}^2u_{21}u_{30}^2 + 288u_{01}u_{02}^2\\
&+ 156u_{01}u_{02}u_{10}u_{31} + 576u_{01}u_{02}u_{11}u_{30} + 144u_{01}u_{02}u_{20}u_{30}^2\\
&- 10u_{01}u_{10}^2u_{31}^2 - 20u_{01}u_{10}u_{11}u_{30}u_{31} + 20u_{01}u_{10}u_{20}u_{21}u_{31}\\
&- 864u_{01}u_{11}^2u_{21} + 72u_{01}u_{11}^2u_{30}^2 + 82u_{01}u_{11}u_{20}^2u_{31}\\
&- 864u_{01}u_{11}u_{20}u_{21}u_{30} + 144u_{01}u_{12}u_{20}^2u_{30} - 96u_{01}u_{20}^3u_{30}u_{31}\\
&+ 144u_{01}u_{20}^2u_{21}^2 - 144u_{01}u_{20}^2u_{21}u_{30}^2 - 12u_{02}u_{10}^2u_{30}u_{31} + 12u_{02}u_{10}u_{20}^2u_{31}\\
&- 288u_{02}u_{11}^2u_{20}\\
&- 72u_{02}u_{11}u_{20}^2u_{30} + 2u_{10}^3u_{22}u_{31} - 6u_{10}^2u_{12}u_{20}u_{31} + 20u_{10}u_{11}^2u_{20}u_{31}\\
&- 36u_{11}^4 - 144u_{11}^3u_{20}u_{30} + 432u_{11}^2u_{20}^2u_{21} - 72u_{11}u_{12}u_{20}^3 + 42u_{11}u_{20}^4u_{31}\\
&+ 72u_{11}u_{20}^3u_{21}u_{30})
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(107) &= \Pi_{yy}u_{30}(Q_{2,12} - 288Q_{0,7}u_{01}u_{10} + 72Q_{1,11}u_{10} + 144Q_{1,8}u_{01} - 65u_{01}^3\\
&- 72u_{01}^2u_{10}u_{30} + 324u_{01}^2u_{20}^2 - 514u_{01}u_{10}^2u_{21} - 566u_{01}u_{10}u_{11}u_{20}\\
&- 72u_{01}u_{10}u_{20}^2u_{30} - 6u_{01}u_{20}^4 + 42u_{02}u_{10}^2u_{20} - 2u_{10}^4u_{41}\\
&+ 12u_{10}^3u_{11}u_{40} - 16u_{10}^3u_{12} + 8u_{10}^3u_{20}u_{31} + 28u_{10}^3u_{21}u_{30}\\
&+ 257u_{10}^2u_{11}^2 - 36u_{10}^2u_{11}u_{20}u_{30} - 24u_{10}^2u_{20}^2u_{21} + 48u_{10}u_{11}u_{20}^3)\\
&+ 2\Pi_{xy}u_{21}(-Q_{2,12} + 288Q_{0,7}u_{01}u_{10} - 72Q_{1,11}u_{10} - 144Q_{1,8}u_{01}\\
&+ 65u_{01}^3 + 72u_{01}^2u_{10}u_{30} - 324u_{01}^2u_{20}^2 + 514u_{01}u_{10}^2u_{21}\\
&+ 566u_{01}u_{10}u_{11}u_{20} + 72u_{01}u_{10}u_{20}^2u_{30} + 6u_{01}u_{20}^4\\
&- 42u_{02}u_{10}^2u_{20} + 2u_{10}^4u_{41} - 12u_{10}^3u_{11}u_{40} + 16u_{10}^3u_{12} - 8u_{10}^3u_{20}u_{31}\\
&- 28u_{10}^3u_{21}u_{30} - 257u_{10}^2u_{11}^2 + 36u_{10}^2u_{11}u_{20}u_{30} + 24u_{10}^2u_{20}^2u_{21}\\
&- 48u_{10}u_{11}u_{20}^3) + \Pi_{xx}u_{12}(Q_{2,12} - 288Q_{0,7}u_{01}u_{10} + 72Q_{1,11}u_{10}\\
&+ 144Q_{1,8}u_{01} - 65u_{01}^3 - 72u_{01}^2u_{10}u_{30} + 324u_{01}^2u_{20}^2 - 514u_{01}u_{10}^2u_{21}\\
&- 566u_{01}u_{10}u_{11}u_{20} - 72u_{01}u_{10}u_{20}^2u_{30} - 6u_{01}u_{20}^4 + 42u_{02}u_{10}^2u_{20}\\
&- 2u_{10}^4u_{41} + 12u_{10}^3u_{11}u_{40} - 16u_{10}^3u_{12} + 8u_{10}^3u_{20}u_{31} + 28u_{10}^3u_{21}u_{30}\\
&+ 257u_{10}^2u_{11}^2 - 36u_{10}^2u_{11}u_{20}u_{30} - 24u_{10}^2u_{20}^2u_{21} + 48u_{10}u_{11}u_{20}^3)\\
&+ 2\Pi_yu_{21}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11}\\
&- 24Q_{0,7}u_{20}^3 + 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13}\\
&+ 144Q_{1,8}u_{11} - 195u_{01}^2u_{11} + 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31}\\
&- 20u_{01}u_{10}u_{11}u_{30} + 20u_{01}u_{10}u_{20}u_{21} + 82u_{01}u_{11}u_{20}^2 - 96u_{01}u_{20}^3u_{30}\\
&- 12u_{02}u_{10}^2u_{30} + 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20}\\
&+ 20u_{10}u_{11}^2u_{20} + 42u_{11}u_{20}^4) + \Pi_xu_{12}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20}\\
&+ 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3 - 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20}\\
&- Q_{1,13} - 144Q_{1,8}u_{11} + 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31}\\
&+ 20u_{01}u_{10}u_{11}u_{30} - 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2\\
&+ 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30} - 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20}\\
&- 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4) + \Pi(-288Q_{0,12}u_{10}u_{22} + 216Q_{0,7}^2u_{12}\\
&+ 288Q_{0,7}u_{01}u_{12}u_{30} - 144Q_{0,7}u_{01}u_{20}u_{22} - 288Q_{0,7}u_{10}u_{11}u_{22}
\end{aligned}
$$

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\begin{aligned}
&- 288Q_{0,7}u_{10}u_{12}u_{21} - 432Q_{0,7}u_{11}u_{12}u_{20} - 72Q_{0,7}u_{12}u_{20}^2u_{30}\\
&- 24Q_{0,7}u_{20}^3u_{22} + 72Q_{0,9}u_{01}u_{22} + 216Q_{0,9}u_{11}u_{12} + 72Q_{0,9}u_{12}u_{20}u_{30}\\
&+ 36Q_{0,9}u_{20}^2u_{22} + 72Q_{1,11}u_{12}u_{30} + 72Q_{1,11}u_{20}u_{22} + Q_{1,13}u_{22}\\
&+ 144Q_{1,8}u_{11}u_{22} + 144Q_{1,8}u_{12}u_{21} - 195u_{01}^2u_{11}u_{22} - 288u_{01}^2u_{12}u_{21}\\
&+ 156u_{01}u_{02}u_{10}u_{22} - 10u_{01}u_{10}^2u_{22}u_{31} - 20u_{01}u_{10}u_{11}u_{22}u_{30}\\
&+ 20u_{01}u_{10}u_{20}u_{21}u_{22} - 432u_{01}u_{11}^2u_{12} - 432u_{01}u_{11}u_{12}u_{20}u_{30}\\
&+ 82u_{01}u_{11}u_{20}^2u_{22} + 144u_{01}u_{12}u_{20}^2u_{21} - 144u_{01}u_{12}u_{20}^2u_{30}^2\\
&- 96u_{01}u_{20}^3u_{22}u_{30} - 12u_{02}u_{10}^2u_{22}u_{30} + 12u_{02}u_{10}u_{20}^2u_{22}\\
&+ 2u_{10}^3u_{22}^2 - 6u_{10}^2u_{12}u_{20}u_{22} + 20u_{10}u_{11}^2u_{20}u_{22}\\
&+ 216u_{11}^2u_{12}u_{20}^2 + 72u_{11}u_{12}u_{20}^3u_{30} + 42u_{11}u_{20}^4u_{22})
\end{aligned}
$$

$$
\begin{aligned}
D_x\Pi(108) &= -\Pi_{yy}x + \Pi_y u_{30} + \Pi u_{31}\\
D_y\Pi(108) &= \Pi_{yy}xu_{30} - 2\Pi_{xy}xu_{21} + \Pi_{xx}xu_{12} + 2\Pi_y u_{21} - \Pi_x u_{12} + \Pi u_{22}\\
D_x\Pi(109) &= \Pi_{yy}xu_{10} - 4\Pi_{yy}u - \Pi_y x(u_{11} + u_{20}u_{30}) + \Pi_y(4u_{01} + 3u_{10}u_{30})\\
&\quad + \Pi x(u_{12} - u_{20}u_{31} - u_{21}u_{30})\\
&\quad + \Pi(-4u_{02} + 3u_{10}u_{31} - 3u_{11}u_{30} + 4u_{20}u_{21})\\
D_y\Pi(109) &= -\Pi_{yy}xu_{10}u_{30} + 4\Pi_{yy}uu_{30} + 2\Pi_{xy}xu_{10}u_{21} - 8\Pi_{xy}uu_{21} - \Pi_{xx}xu_{10}u_{12}\\
&\quad + 4\Pi_{xx}uu_{12} - 2\Pi_y xu_{20}u_{21} + 6\Pi_y u_{10}u_{21} + \Pi_x xu_{12}u_{20} - 3\Pi_x u_{10}u_{12}\\
&\quad - \Pi x(u_{12}u_{30} + u_{20}u_{22}) + \Pi(3u_{10}u_{22} + 2u_{12}u_{20})\\
D_x\Pi(110) &= (4\Pi_{yy}xu_{01} - \Pi_{yy}Q_{1,3} - 4\Pi_y x(u_{02} + u_{11}u_{30})\\
&\quad + \Pi_y(2Q_{0,7} - 2u_{11}u_{20} - u_{20}^2u_{30}) - 4\Pi x(u_{11}u_{31} + u_{21}^2)\\
&\quad + \Pi(-2u_{01}u_{31} + 2u_{02}u_{30} - 12u_{11}u_{21} + 2u_{12}u_{20} - u_{20}^2u_{31}\\
&\quad - 2u_{20}u_{21}u_{30}))/4\\
D_y\Pi(110) &= (-4\Pi_{yy}xu_{01}u_{30} + \Pi_{yy}Q_{1,3}u_{30} + 8\Pi_{xy}xu_{01}u_{21} - 2\Pi_{xy}Q_{1,3}u_{21}\\
&\quad - 4\Pi_{xx}xu_{01}u_{12} + \Pi_{xx}Q_{1,3}u_{12} - 8\Pi_y xu_{11}u_{21} + 2\Pi_y u_{21}(-2u_{01} - u_{20}^2)\\
&\quad + 4\Pi_x xu_{11}u_{12} + \Pi_x u_{12}(2u_{01} + u_{20}^2) - 4\Pi x(u_{11}u_{22} + u_{12}u_{21})\\
&\quad + \Pi(-2u_{01}u_{22} - 6u_{11}u_{12} - 2u_{12}u_{20}u_{30} - u_{20}^2u_{22}))/4\\
D_x\Pi(111) &= (\Pi_{yy}x(Q_{2,5} + 8u_{01}u_{10}) + 3\Pi_{yy}(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01})\\
&\quad + \Pi_y x(3Q_{0,9} + 18Q_{1,6}u_{30} - 6u_{01}u_{11} - 6u_{01}u_{20}u_{30} - 12u_{02}u_{10}\\
&\quad - 12u_{10}u_{11}u_{30} - u_{20}^3u_{30}) + \Pi_y(7Q_{2,5}u_{30} + 30Q_{0,7}u_{10} + 15Q_{1,3}u_{11}\\
&\quad + 15Q_{1,3}u_{20}u_{30} - 24Q_{1,8} + 48uu_{02} + 48uu_{11}u_{30} - 28u_{01}u_{10}u_{30}\\
&\quad - 24u_{01}u_{20}^2 + 18u_{10}u_{11}u_{20} + 9u_{10}u_{20}^2u_{30}) + \Pi x(36Q_{0,7}u_{21} + 18Q_{1,6}u_{31}\\
&\quad + 6u_{01}u_{12} - 6u_{01}u_{20}u_{31} + 30u_{01}u_{21}u_{30} + 12u_{02}u_{11}\\
&\quad + 6u_{02}u_{20}u_{30} - 12u_{10}u_{11}u_{31} - 12u_{10}u_{21}^2 + 3u_{11}^2u_{30} - 36u_{11}u_{20}u_{21}\\
&\quad + 3u_{12}u_{20}^2 - u_{20}^3u_{31} - 3u_{20}^2u_{21}u_{30}) + \Pi(7Q_{2,5}u_{31} + 96Q_{0,12} - 60Q_{0,7}u_{11}\\
&\quad - 30Q_{0,7}u_{20}u_{30} + 21Q_{0,9}u_{30} - 15Q_{1,3}u_{12} + 15Q_{1,3}u_{20}u_{31}\\
&\quad + 15Q_{1,3}u_{21}u_{30} - 216Q_{1,6}u_{21} + 48uu_{11}u_{31} + 48uu_{21}^2 - 48u_{01}u_{02}\\
&\quad - 10u_{01}u_{10}u_{31} - 84u_{01}u_{11}u_{30} - 30u_{01}u_{20}u_{30}^2 - 18u_{02}u_{10}u_{30}\\
&\quad + 108u_{10}u_{11}u_{21} - 18u_{10}u_{12}u_{20} + 9u_{10}u_{20}^2u_{31} + 18u_{10}u_{20}u_{21}u_{30}\\
&\quad + 60u_{11}^2u_{20} + 21u_{11}u_{20}^2u_{30}))/3\\
D_y\Pi(111) &= (\Pi_{yy}xu_{30}(-Q_{2,5} - 8u_{01}u_{10}) + 3\Pi_{yy}u_{30}(8Q_{3,4} - 3Q_{1,3}u_{10} + 16uu_{01})\\
&\quad + 2\Pi_{xy}xu_{21}(Q_{2,5} + 8u_{01}u_{10}) + 6\Pi_{xy}u_{21}(-8Q_{3,4} + 3Q_{1,3}u_{10} - 16uu_{01})\\
&\quad + \Pi_{xx}xu_{12}(-Q_{2,5} - 8u_{01}u_{10}) + 3\Pi_{xx}u_{12}(8Q_{3,4} - 3Q_{1,3}u_{10} + 16uu_{01})
\end{aligned}
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\begin{aligned}
&+ 2\Pi_y x u_{21}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&+ 2\Pi_y u_{21}(7Q_{2,5} + 15Q_{1,3}u_{20} + 48uu_{11} - 10u_{01}u_{10} + 9u_{10}u_{20}^2)\\
&+ \Pi_x x u_{12}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&+ \Pi_x u_{12}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 48uu_{11} + 10u_{01}u_{10} - 9u_{10}u_{20}^2)\\
&+ \Pi x(18Q_{0,7}u_{12} + 18Q_{1,6}u_{22} + 12u_{01}u_{12}u_{30} - 6u_{01}u_{20}u_{22}\\
&- 12u_{10}u_{11}u_{22} - 12u_{10}u_{12}u_{21} - 18u_{11}u_{12}u_{20} - 3u_{12}u_{20}^2u_{30} - u_{20}^3u_{22})\\
&+ \Pi(7Q_{2,5}u_{22} + 15Q_{1,3}u_{12}u_{30} + 15Q_{1,3}u_{20}u_{22} - 108Q_{1,6}u_{12}\\
&+ 48uu_{11}u_{22} + 48uu_{12}u_{21} - 10u_{01}u_{10}u_{22} + 54u_{10}u_{11}u_{12}\\
&+ 18u_{10}u_{12}u_{20}u_{30} + 9u_{10}u_{20}^2u_{22}))/3\\
D_x\Pi(112) &= (120\Pi_{yy}x(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 2\Pi_{yy}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 120\Pi_y x(4Q_{0,12} - 2Q_{0,7}u_{11} - 2Q_{0,7}u_{20}u_{30} + Q_{0,9}u_{30} - 4u_{01}u_{02}\\
&- 6u_{01}u_{11}u_{30} - 2u_{01}u_{20}u_{30}^2 + 2u_{11}^2u_{20} + u_{11}u_{20}^2u_{30})\\
&+ 5\Pi_y(-24Q_{0,7}u_{10}u_{30} - 12Q_{1,11} + 24Q_{1,3}u_{02} + 24Q_{1,3}u_{11}u_{30}\\
&+ 72Q_{1,6}u_{11} + 72Q_{1,6}u_{20}u_{30} + 12Q_{1,8}u_{30} - 24u_{01}u_{11}u_{20}\\
&+ 12u_{01}u_{20}^2u_{30} - 4u_{11}u_{20}^3 - u_{20}^4u_{30}) + 120\Pi x(2Q_{0,7}u_{12} - 2Q_{0,7}u_{20}u_{31}\\
&- 2Q_{0,7}u_{21}u_{30} + Q_{0,9}u_{31} - 4u_{01}u_{11}u_{31} + 2u_{01}u_{12}u_{30}\\
&- 2u_{01}u_{20}u_{30}u_{31} - 4u_{01}u_{21}^2 - 2u_{01}u_{21}u_{30}^2 + 2u_{02}^2 + 2u_{02}u_{11}u_{30}\\
&- 6u_{11}^2u_{21} + u_{11}u_{20}^2u_{31}) + 5\Pi(-48Q_{0,12}u_{30} - 48Q_{0,7}u_{02}\\
&- 24Q_{0,7}u_{10}u_{31} - 24Q_{0,7}u_{11}u_{30} + 72Q_{0,9}u_{21} + 24Q_{1,3}u_{11}u_{31}\\
&+ 24Q_{1,3}u_{21}^2 - 72Q_{1,6}u_{12} + 72Q_{1,6}u_{20}u_{31} + 72Q_{1,6}u_{21}u_{30} + 12Q_{1,8}u_{31}\\
&- 12u_{01}^2u_{31} - 24u_{01}u_{02}u_{30} - 144u_{01}u_{11}u_{21} - 24u_{01}u_{11}u_{30}^2\\
&+ 24u_{01}u_{12}u_{20} - 24u_{01}u_{20}u_{21}u_{30} + 48u_{02}u_{11}u_{20} + 12u_{02}u_{20}^2u_{30} - 24u_{11}^3\\
&- 12u_{11}^2u_{20}u_{30} + 4u_{12}u_{20}^3 - u_{20}^4u_{31} - 4u_{20}^3u_{21}u_{30}))/120\\
D_y\Pi(112) &= (120\Pi_{yy}xu_{30}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ 2\Pi_{yy}u_{30}(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)\\
&+ 240\Pi_{xy}xu_{21}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 4\Pi_{xy}u_{21}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 120\Pi_{xx}xu_{12}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ 2\Pi_{xx}u_{12}(3Q_{2,7} + 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3)\\
&+ 240\Pi_y xu_{21}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 10\Pi_y u_{21}(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8} - 12u_{01}^2\\
&- u_{20}^4) + 120\Pi_x xu_{12}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ 5\Pi_x u_{12}(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)\\
&+ 120\Pi x(-2Q_{0,7}u_{12}u_{30} - 2Q_{0,7}u_{20}u_{22} + Q_{0,9}u_{22} - 4u_{01}u_{11}u_{22}\\
&- 4u_{01}u_{12}u_{21} - 2u_{01}u_{12}u_{30}^2 - 2u_{01}u_{20}u_{22}u_{30} - 3u_{11}^2u_{12} + u_{11}u_{20}^2u_{22})\\
&+ 5\Pi(-24Q_{0,7}u_{10}u_{22} + 36Q_{0,9}u_{12} + 24Q_{1,3}u_{11}u_{22} + 24Q_{1,3}u_{12}u_{21}\\
&+ 72Q_{1,6}u_{12}u_{30} + 72Q_{1,6}u_{20}u_{22} + 12Q_{1,8}u_{22} - 12u_{01}^2u_{22} - 72u_{01}u_{11}u_{12}\\
&- 24u_{01}u_{12}u_{20}u_{30} - 4u_{12}u_{20}^3u_{30} - u_{20}^4u_{22}))/120\\
D_x\Pi(113) &= -\Pi_{yy}y + \Pi_y\\
D_y\Pi(113) &= \Pi_{yy}yu_{30} - 2\Pi_{xy}yu_{21} + \Pi_{xx}yu_{12}
\end{aligned}
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\begin{aligned}
D_x\Pi(114) &= \Pi_{yy}yu_{01} + \Pi_{yy}u - \Pi_y y(u_{02} + u_{11}u_{30}) + \Pi_y(-2u_{01} - u_{10}u_{30}) \\
&\quad - \Pi y(u_{11}u_{31} + u_{21}^2) + \Pi(3u_{02} - u_{10}u_{31} + 2u_{11}u_{30} - 2u_{20}u_{21}) \\
D_y\Pi(114) &= -\Pi_{yy}yu_{01}u_{30} - \Pi_{yy}uu_{30} + 2\Pi_{xy}yu_{01}u_{21} + 2\Pi_{xy}uu_{21} \\
&\quad - \Pi_{xx}yu_{01}u_{12} - \Pi_{xx}uu_{12} - 2\Pi_y yu_{11}u_{21} - 2\Pi_y u_{10}u_{21} \\
&\quad + \Pi_x yu_{11}u_{12} + \Pi_x u_{10}u_{12} - \Pi y(u_{11}u_{22} + u_{12}u_{21}) - \Pi(u_{10}u_{22} + u_{12}u_{20}) \\
D_x\Pi(115) &= \Pi_{yy}y(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2) + \Pi_{yy}(3Q_{3,4} - Q_{1,3}u_{10} \\
&\quad + 4uu_{01}) + \Pi_y y(4Q_{0,12} - 2Q_{0,7}u_{11} - 2Q_{0,7}u_{20}u_{30} + Q_{0,9}u_{30} - 4u_{01}u_{02} \\
&\quad - 6u_{01}u_{11}u_{30} - 2u_{01}u_{20}u_{30}^2 + 2u_{11}^2u_{20} + u_{11}u_{20}^2u_{30}) \\
&\quad + \Pi_y(-Q_{2,5}u_{30} - 6Q_{0,7}u_{10} - 2Q_{1,3}u_{11} - 2Q_{1,3}u_{20}u_{30} + 4Q_{1,8} - 4uu_{02} \\
&\quad - 4uu_{11}u_{30} + 4u_{01}u_{10}u_{30} + 4u_{01}u_{20}^2 - 2u_{10}u_{11}u_{20} - u_{10}u_{20}^2u_{30}) \\
&\quad + \Pi y(2Q_{0,7}u_{12} - 2Q_{0,7}u_{20}u_{31} - 2Q_{0,7}u_{21}u_{30} + Q_{0,9}u_{31} - 4u_{01}u_{11}u_{31} \\
&\quad + 2u_{01}u_{12}u_{30} - 2u_{01}u_{20}u_{30}u_{31} - 4u_{01}u_{21}^2 - 2u_{01}u_{21}u_{30}^2 \\
&\quad + 2u_{02}^2 + 2u_{02}u_{11}u_{30} - 6u_{11}^2u_{21} + u_{11}u_{20}^2u_{31}) \\
&\quad + \Pi(-Q_{2,5}u_{31} - 20Q_{0,12} + 12Q_{0,7}u_{11} + 6Q_{0,7}u_{20}u_{30} - 4Q_{0,9}u_{30} \\
&\quad + 2Q_{1,3}u_{12} - 2Q_{1,3}u_{20}u_{31} - 2Q_{1,3}u_{21}u_{30} + 36Q_{1,6}u_{21} - 4uu_{11}u_{31} \\
&\quad - 4uu_{21}^2 + 8u_{01}u_{02} + 2u_{01}u_{10}u_{31} + 16u_{01}u_{11}u_{30} + 6u_{01}u_{20}u_{30}^2 \\
&\quad + 2u_{02}u_{10}u_{30} - 12u_{10}u_{11}u_{21} + 2u_{10}u_{12}u_{20} - u_{10}u_{20}^2u_{31} \\
&\quad - 2u_{10}u_{20}u_{21}u_{30} - 12u_{11}^2u_{20} - 4u_{11}u_{20}^2u_{30}) \\
D_y\Pi(115) &= \Pi_{yy}yu_{30}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) \\
&\quad + \Pi_{yy}u_{30}(-3Q_{3,4} + Q_{1,3}u_{10} - 4uu_{01}) \\
&\quad + 2\Pi_{xy}yu_{21}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2) \\
&\quad + 2\Pi_{xy}u_{21}(3Q_{3,4} - Q_{1,3}u_{10} + 4uu_{01}) \\
&\quad + \Pi_{xx}yu_{12}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2) \\
&\quad + \Pi_{xx}u_{12}(-3Q_{3,4} + Q_{1,3}u_{10} - 4uu_{01}) \\
&\quad + 2\Pi_y yu_{21}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2) \\
&\quad + 2\Pi_y u_{21}(-Q_{2,5} - 2Q_{1,3}u_{20} - 4uu_{11} + 2u_{01}u_{10} - u_{10}u_{20}^2) \\
&\quad + \Pi_x yu_{12}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2) \\
&\quad + \Pi_x u_{12}(Q_{2,5} + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10} + u_{10}u_{20}^2) \\
&\quad + \Pi y(-2Q_{0,7}u_{12}u_{30} - 2Q_{0,7}u_{20}u_{22} + Q_{0,9}u_{22} - 4u_{01}u_{11}u_{22} \\
&\quad - 4u_{01}u_{12}u_{21} - 2u_{01}u_{12}u_{30}^2 - 2u_{01}u_{20}u_{22}u_{30} - 3u_{11}^2u_{12} + u_{11}u_{20}^2u_{22}) \\
&\quad + \Pi(-Q_{2,5}u_{22} - 2Q_{1,3}u_{12}u_{30} - 2Q_{1,3}u_{20}u_{22} + 18Q_{1,6}u_{12} - 4uu_{11}u_{22} \\
&\quad - 4uu_{12}u_{21} + 2u_{01}u_{10}u_{22} - 6u_{10}u_{11}u_{12} - 2u_{10}u_{12}u_{20}u_{30} - u_{10}u_{20}^2u_{22}) \\
D_x\Pi(116) &= (\Pi_{yy}y^2 - 2\Pi_y y + 2\Pi)/2 \\
D_y\Pi(116) &= (-\Pi_{yy}y^2u_{30} + 2\Pi_{xy}y^2u_{21} - \Pi_{xx}y^2u_{12})/2 \\
D_x\Pi(117) &= \Pi_{yy}xy - \Pi_y x - \Pi_y yu_{30} - \Pi yu_{31} + \Pi u_{30} \\
D_y\Pi(117) &= -\Pi_{yy}xyu_{30} + 2\Pi_{xy}xyu_{21} - \Pi_{xx}xyu_{12} - 2\Pi_y yu_{21} \\
&\quad + \Pi_x yu_{12} - \Pi yu_{22} \\
D_x\Pi(118) &= (\Pi_{yy}x^2 - 2\Pi_y xu_{30} - 2\Pi xu_{31} - 4\Pi u_{21})/2 \\
D_y\Pi(118) &= (-\Pi_{yy}x^2u_{30} + 2\Pi_{xy}x^2u_{21} - \Pi_{xx}x^2u_{12} - 4\Pi_y xu_{21} \\
&\quad + 2\Pi_x xu_{12} - 2\Pi xu_{22} - 2\Pi u_{12})/2 \\
D_x\Pi(119) &= (-\Pi_{yy}x^2u_{10} - 4\Pi_{yy}xyu_{01} + 4\Pi_{yy}xu + \Pi_{yy}yQ_{1,3} \\
&\quad + \Pi_y x^2(u_{11} + u_{20}u_{30}) + 4\Pi_y xy(u_{02} + u_{11}u_{30}) - 2\Pi_y xu_{10}u_{30}
\end{aligned}
$$

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\begin{aligned}
&+ \Pi_y y(-2Q_{0,7} + 2u_{11}u_{20} + u_{20}^2 u_{30})\\
&+ \Pi_y(-Q_{1,3} - 4uu_{30}) + \Pi x^2(-u_{12} + u_{20}u_{31} + u_{21}u_{30})\\
&+ 4\Pi xy(u_{11}u_{31} + u_{21}^2) + 2\Pi x(-2u_{02} - u_{10}u_{31} - u_{11}u_{30})\\
&+ \Pi y(2u_{01}u_{31} - 2u_{02}u_{30} + 12u_{11}u_{21} - 2u_{12}u_{20} + u_{20}^2u_{31} + 2u_{20}u_{21}u_{30})\\
&+ \Pi(4Q_{0,7} - 4uu_{31} + 6u_{01}u_{30} - 12u_{10}u_{21} - 4u_{11}u_{20} - u_{20}^2u_{30}))/4\\
D_y\Pi(119) &= (\Pi_{yy}x^2u_{10}u_{30} + 4\Pi_{yy}xyu_{01}u_{30} - 4\Pi_{yy}xuu_{30}\\
&- \Pi_{yy}yQ_{1,3}u_{30} - 2\Pi_{xy}x^2u_{10}u_{21} - 8\Pi_{xy}xyu_{01}u_{21} + 8\Pi_{xy}xuu_{21}\\
&+ 2\Pi_{xy}yQ_{1,3}u_{21} + \Pi_{xx}x^2u_{10}u_{12} + 4\Pi_{xx}xyu_{01}u_{12} - 4\Pi_{xx}xuu_{12}\\
&- \Pi_{xx}yQ_{1,3}u_{12} + 2\Pi_y x^2u_{20}u_{21} + 8\Pi_y xyu_{11}u_{21} - 4\Pi_y xu_{10}u_{21}\\
&+ 2\Pi_y yu_{21}(2u_{01} + u_{20}^2) - 8\Pi_y uu_{21} - \Pi_x x^2u_{12}u_{20} - 4\Pi_x xyu_{11}u_{12}\\
&+ 2\Pi_x xu_{10}u_{12} + \Pi_x yu_{12}(-2u_{01} - u_{20}^2) + 4\Pi_x uu_{12}\\
&+ \Pi x^2(u_{12}u_{30} + u_{20}u_{22}) + 4\Pi xy(u_{11}u_{22} + u_{12}u_{21})\\
&- 2\Pi xu_{10}u_{22} + \Pi y(2u_{01}u_{22} + 6u_{11}u_{12} + 2u_{12}u_{20}u_{30} + u_{20}^2u_{22})\\
&+ 2\Pi(-2uu_{22} - 3u_{10}u_{12}))/4\\
D_x\Pi(120) &= (2\Pi(101)u_{10}u_{20} - 2\Pi_{yy}x^2u_{01} + \Pi_{yy}xQ_{1,3}\\
&+ 2\Pi_{yy}u_{10}^2 + 2\Pi_y x^2(u_{02} + u_{11}u_{30}) + \Pi_y x(-2Q_{0,7} + 2u_{11}u_{20} + u_{20}^2u_{30})\\
&+ \Pi_y(-Q_{1,3}u_{30} - 2u_{10}u_{11} - 2u_{10}u_{20}u_{30}) + 2\Pi x^2(u_{11}u_{31} + u_{21}^2)\\
&+ \Pi x(2u_{01}u_{31} - 2u_{02}u_{30} + 12u_{11}u_{21} - 2u_{12}u_{20} + u_{20}^2u_{31} + 2u_{20}u_{21}u_{30})\\
&+ \Pi(2Q_{0,7}u_{30} - Q_{1,3}u_{31} + 2u_{01}u_{30}^2 + 2u_{10}u_{12} - 2u_{10}u_{20}u_{31}\\
&- 2u_{10}u_{21}u_{30} + 2u_{11}^2))/4\\
D_y\Pi(120) &= (2\Pi(101)u_{10}u_{11} + 2\Pi_{yy}x^2u_{01}u_{30} - \Pi_{yy}xQ_{1,3}u_{30} - 2\Pi_{yy}u_{10}^2u_{30}\\
&- 4\Pi_{xy}x^2u_{01}u_{21} + 2\Pi_{xy}xQ_{1,3}u_{21} + 4\Pi_{xy}u_{10}^2u_{21} + 2\Pi_{xx}x^2u_{01}u_{12}\\
&- \Pi_{xx}xQ_{1,3}u_{12} - 2\Pi_{xx}u_{10}^2u_{12} + 4\Pi_y x^2u_{11}u_{21}\\
&+ 2\Pi_y xu_{21}(2u_{01} + u_{20}^2) + 2\Pi_y u_{21}(-Q_{1,3} - 2u_{10}u_{20}) - 2\Pi_x x^2u_{11}u_{12}\\
&+ \Pi_x xu_{12}(-2u_{01} - u_{20}^2) + \Pi_x u_{12}(Q_{1,3} + 2u_{10}u_{20})\\
&+ 2\Pi x^2(u_{11}u_{22} + u_{12}u_{21})\\
&+ \Pi x(2u_{01}u_{22} + 6u_{11}u_{12} + 2u_{12}u_{20}u_{30} + u_{20}^2u_{22})\\
&+ \Pi(-Q_{1,3}u_{22} - 2u_{10}u_{12}u_{30} - 2u_{10}u_{20}u_{22}))/4\\
D_x\Pi(121) &= (10\Pi_{yy}x^2(-Q_{2,5} - 8u_{01}u_{10}) + 120\Pi_{yy}xy(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2\\
&+ u_{01}u_{20}^2) + 60\Pi_{yy}x(2Q_{3,4} - Q_{1,3}u_{10} + 8uu_{01}) + 2\Pi_{yy}y(3Q_{2,7}\\
&+ 60Q_{1,3}u_{01} + 180Q_{1,6}u_{10} - u_{10}u_{20}^3) + 60\Pi_{yy}(-2Q_{1,3}u - u_{10}^3)\\
&+ 10\Pi_y x^2(-3Q_{0,9} - 18Q_{1,6}u_{30} + 6u_{01}u_{11} + 6u_{01}u_{20}u_{30} + 12u_{02}u_{10}\\
&+ 12u_{10}u_{11}u_{30} + u_{20}^3u_{30}) + 120\Pi_y xy(-4Q_{0,12} + 2Q_{0,7}u_{11} + 2Q_{0,7}u_{20}u_{30}\\
&- Q_{0,9}u_{30} + 4u_{01}u_{02} + 6u_{01}u_{11}u_{30} + 2u_{01}u_{20}u_{30}^2 - 2u_{11}^2u_{20} - u_{11}u_{20}^2u_{30})\\
&+ 20\Pi_y x(-Q_{2,5}u_{30} + 6Q_{0,7}u_{10} - 3Q_{1,3}u_{11} - 3Q_{1,3}u_{20}u_{30} - 24uu_{02}\\
&- 24uu_{11}u_{30} + 4u_{01}u_{10}u_{30} - 6u_{10}u_{11}u_{20} - 3u_{10}u_{20}^2u_{30})\\
&+ 5\Pi_y y(24Q_{0,7}u_{10}u_{30} + 12Q_{1,11} - 24Q_{1,3}u_{02} - 24Q_{1,3}u_{11}u_{30}\\
&- 72Q_{1,6}u_{11} - 72Q_{1,6}u_{20}u_{30} - 12Q_{1,8}u_{30} + 24u_{01}u_{11}u_{20} - 12u_{01}u_{20}^2u_{30}\\
&+ 4u_{11}u_{20}^3 + u_{20}^4u_{30}) + 2\Pi_y(-3Q_{2,7} - 60Q_{3,4}u_{30} + 120Q_{0,7}u\\
&+ 90Q_{1,3}u_{10}u_{30} - 180Q_{1,6}u_{10} - 120uu_{11}u_{20} - 60uu_{20}^2u_{30} + 90u_{10}^2u_{11}
\end{aligned}
$$

$$
\begin{aligned}
&+ 90u_{10}^2u_{20}u_{30} + u_{10}u_{20}^3) + 10\Pi x^2(-36Q_{0,7}u_{21} - 18Q_{1,6}u_{31} - 6u_{01}u_{12}\\
&+ 6u_{01}u_{20}u_{31} - 30u_{01}u_{21}u_{30} - 12u_{02}u_{11} - 6u_{02}u_{20}u_{30} + 12u_{10}u_{11}u_{31}\\
&+ 12u_{10}u_{21}^2 - 3u_{11}^2u_{30} + 36u_{11}u_{20}u_{21} - 3u_{12}u_{20}^2 + u_{20}^3u_{31} + 3u_{20}^2u_{21}u_{30})\\
&+ 120\Pi xy(-2Q_{0,7}u_{12} + 2Q_{0,7}u_{20}u_{31} + 2Q_{0,7}u_{21}u_{30} - Q_{0,9}u_{31}\\
&+ 4u_{01}u_{11}u_{31} - 2u_{01}u_{12}u_{30} + 2u_{01}u_{20}u_{30}u_{31} + 4u_{01}u_{21}^2 + 2u_{01}u_{21}u_{30}^2\\
&- 2u_{02}^2 - 2u_{02}u_{11}u_{30} + 6u_{11}^2u_{21} - u_{11}u_{20}^2u_{31}) + 20\Pi x(-Q_{2,5}u_{31} + 24Q_{0,12}\\
&- 12Q_{0,7}u_{11} - 6Q_{0,7}u_{20}u_{30} + 3Q_{0,9}u_{30} + 3Q_{1,3}u_{12} - 3Q_{1,3}u_{20}u_{31}\\
&- 3Q_{1,3}u_{21}u_{30} - 24uu_{11}u_{31} - 24uu_{21}^2 - 2u_{01}u_{10}u_{31} - 12u_{01}u_{11}u_{30}\\
&- 6u_{01}u_{20}u_{30}^2 + 6u_{02}u_{10}u_{30} - 36u_{10}u_{11}u_{21} + 6u_{10}u_{12}u_{20} - 3u_{10}u_{20}^2u_{31}\\
&- 6u_{10}u_{20}u_{21}u_{30} + 12u_{11}^2u_{20} + 3u_{11}u_{20}^2u_{30})\\
&+ 5\Pi y(48Q_{0,12}u_{30} + 48Q_{0,7}u_{02} + 24Q_{0,7}u_{10}u_{31} + 24Q_{0,7}u_{11}u_{30}\\
&- 72Q_{0,9}u_{21} - 24Q_{1,3}u_{11}u_{31} - 24Q_{1,3}u_{21}^2 + 72Q_{1,6}u_{12} - 72Q_{1,6}u_{20}u_{31}\\
&- 72Q_{1,6}u_{21}u_{30} - 12Q_{1,8}u_{31} + 12u_{01}^2u_{31} + 24u_{01}u_{02}u_{30} + 144u_{01}u_{11}u_{21}\\
&+ 24u_{01}u_{11}u_{30}^2 - 24u_{01}u_{12}u_{20} + 24u_{01}u_{20}u_{21}u_{30} - 48u_{02}u_{11}u_{20}\\
&- 12u_{02}u_{20}^2u_{30} + 24u_{11}^3 + 12u_{11}^2u_{20}u_{30} - 4u_{12}u_{20}^3 + u_{20}^4u_{31} + 4u_{20}^3u_{21}u_{30})\\
&+ 5\Pi(-24Q_{2,5}u_{21} - 24Q_{3,4}u_{31} - 96Q_{0,7}u_{01} - 48Q_{0,7}u_{10}u_{30} - 24Q_{1,11}\\
&+ 24Q_{1,3}u_{02} + 36Q_{1,3}u_{10}u_{31} + 12Q_{1,3}u_{11}u_{30} + 144Q_{1,6}u_{11}\\
&+ 72Q_{1,6}u_{20}u_{30} - 12Q_{1,8}u_{30} - 48uu_{01}u_{31} + 48uu_{02}u_{30} - 288uu_{11}u_{21}\\
&+ 48uu_{12}u_{20} - 24uu_{20}^2u_{31} - 48uu_{20}u_{21}u_{30} - 84u_{01}^2u_{30}\\
&+ 96u_{01}u_{10}u_{21} - 72u_{01}u_{10}u_{30}^2 + 48u_{01}u_{11}u_{20} + 24u_{01}u_{20}^2u_{30} - 36u_{10}^2u_{12}\\
&+ 36u_{10}^2u_{20}u_{31} + 36u_{10}^2u_{21}u_{30} - 72u_{10}u_{11}^2 - 8u_{11}u_{20}^3 - u_{20}^4u_{30}))/120
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(121) &= (10\Pi_{yy}x^2u_{30}(Q_{2,5} + 8u_{01}u_{10}) + 120\Pi_{yy}xyu_{30}(2Q_{0,7}u_{10}\\
&- Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2) + 60\Pi_{yy}xu_{30}(-2Q_{3,4} + Q_{1,3}u_{10} - 8uu_{01})\\
&+ 2\Pi_{yy}yu_{30}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 60\Pi_{yy}u_{30}(2Q_{1,3}u + u_{10}^3) + 20\Pi_{xy}x^2u_{21}(-Q_{2,5} - 8u_{01}u_{10})\\
&+ 240\Pi_{xy}xyu_{21}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ 120\Pi_{xy}xu_{21}(2Q_{3,4} - Q_{1,3}u_{10} + 8uu_{01}) + 4\Pi_{xy}yu_{21}(3Q_{2,7} + 60Q_{1,3}u_{01}\\
&+ 180Q_{1,6}u_{10} - u_{10}u_{20}^3) + 120\Pi_{xy}u_{21}(-2Q_{1,3}u - u_{10}^3)\\
&+ 10\Pi_{xx}x^2u_{12}(Q_{2,5} + 8u_{01}u_{10})\\
&+ 120\Pi_{xx}xyu_{12}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 60\Pi_{xx}xu_{12}(-2Q_{3,4} + Q_{1,3}u_{10} - 8uu_{01})\\
&+ 2\Pi_{xx}yu_{12}(-3Q_{2,7} - 60Q_{1,3}u_{01} - 180Q_{1,6}u_{10} + u_{10}u_{20}^3)\\
&+ 60\Pi_{xx}u_{12}(2Q_{1,3}u + u_{10}^3)\\
&+ 20\Pi_yx^2u_{21}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&+ 240\Pi_yxyu_{21}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ 40\Pi_yxu_{21}(-Q_{2,5} - 3Q_{1,3}u_{20} - 24uu_{11} - 2u_{01}u_{10} - 3u_{10}u_{20}^2)\\
&+ 10\Pi_yyu_{21}(24Q_{0,7}u_{10} - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20} - 12Q_{1,8} + 12u_{01}^2 + u_{20}^4)\\
&+ 120\Pi_yu_{21}(-2Q_{3,4} + 3Q_{1,3}u_{10} - 4uu_{01} - 2uu_{20}^2 + 3u_{10}^2u_{20})\\
&+ 10\Pi_xx^2u_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&+ 120\Pi_xxyu_{12}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 20\Pi_xxu_{12}(Q_{2,5} + 3Q_{1,3}u_{20} + 24uu_{11} + 2u_{01}u_{10} + 3u_{10}u_{20}^2)
\end{aligned}
$$

$$
\begin{aligned}
&+ 5\Pi_x y u_{12}(-24Q_{0,7}u_{10} + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8} - 12u_{01}^2 - u_{20}^4)\\
&+ 60\Pi_x u_{12}(2Q_{3,4} - 3Q_{1,3}u_{10} + 4uu_{01} + 2uu_{20}^2 - 3u_{10}^2 u_{20})\\
&+ 10\Pi x^2(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22}\\
&+ 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22})\\
&+ 120\Pi xy(2Q_{0,7}u_{12}u_{30} + 2Q_{0,7}u_{20}u_{22} - Q_{0,9}u_{22} + 4u_{01}u_{11}u_{22}\\
&+ 4u_{01}u_{12}u_{21} + 2u_{01}u_{12}u_{30}^2 + 2u_{01}u_{20}u_{22}u_{30} + 3u_{11}^2u_{12} - u_{11}u_{20}^2u_{22})\\
&+ 20\Pi x(-Q_{2,5}u_{22} - 3Q_{1,3}u_{12}u_{30} - 3Q_{1,3}u_{20}u_{22} - 24uu_{11}u_{22}\\
&- 24uu_{12}u_{21} - 2u_{01}u_{10}u_{22} - 18u_{10}u_{11}u_{12} - 6u_{10}u_{12}u_{20}u_{30}\\
&- 3u_{10}u_{20}^2u_{22}) + 5\Pi y(24Q_{0,7}u_{10}u_{22} - 36Q_{0,9}u_{12} - 24Q_{1,3}u_{11}u_{22}\\
&- 24Q_{1,3}u_{12}u_{21} - 72Q_{1,6}u_{12}u_{30} - 72Q_{1,6}u_{20}u_{22} - 12Q_{1,8}u_{22}\\
&+ 12u_{01}^2u_{22} + 72u_{01}u_{11}u_{12} + 24u_{01}u_{12}u_{20}u_{30} + 4u_{12}u_{20}^3u_{30} + u_{20}^4u_{22})\\
&+ 60\Pi(-Q_{2,5}u_{12} - 2Q_{3,4}u_{22} + 3Q_{1,3}u_{10}u_{22} - 4uu_{01}u_{22} - 12uu_{11}u_{12}\\
&- 4uu_{12}u_{20}u_{30} - 2uu_{20}^2u_{22} + 4u_{01}u_{10}u_{12} + 3u_{10}^2u_{12}u_{30}\\
&+ 3u_{10}^2u_{20}u_{22}))/120
\end{aligned}
$$

$$
\begin{aligned}
D_x\Pi(122) &= (-\Pi_{yy}x^3 + 2\Pi_{yy}yu_{10} + 3\Pi_y x^2u_{30} - 2\Pi_y y(u_{11} + u_{20}u_{30})\\
&- 2\Pi_y u_{10} + 3\Pi x^2u_{31} + 12\Pi xu_{21} + 2\Pi y(u_{12} - u_{20}u_{31} - u_{21}u_{30})\\
&+ 2\Pi(2u_{11} + u_{20}u_{30}))/2
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(122) &= (\Pi_{yy}x^3u_{30} - 2\Pi_{yy}yu_{10}u_{30} - 2\Pi_{xy}x^3u_{21} + 4\Pi_{xy}yu_{10}u_{21}\\
&+ \Pi_{xx}x^3u_{12} - 2\Pi_{xx}yu_{10}u_{12} + 6\Pi_y x^2u_{21} - 4\Pi_y yu_{20}u_{21}\\
&- 3\Pi_x x^2u_{12} + 2\Pi_x yu_{12}u_{20} + 3\Pi x^2u_{22} + 6\Pi xu_{12}\\
&- 2\Pi y(u_{12}u_{30} + u_{20}u_{22}))/2
\end{aligned}
$$

$$
\begin{aligned}
D_x\Pi(123) &= (12\Pi_{yy}x^3u_{01} - 9\Pi_{yy}x^2Q_{1,3} - 18\Pi_{yy}xu_{10}^2\\
&+ 2\Pi_{yy}y(-Q_{2,5} - 8u_{01}u_{10}) + 24\Pi_{yy}uu_{10} - 12\Pi_y x^3(u_{02} + u_{11}u_{30})\\
&+ 9\Pi_y x^2(2Q_{0,7} - 2u_{11}u_{20} - u_{20}^2u_{30})\\
&+ 18\Pi_y x(Q_{1,3}u_{30} + 2u_{10}u_{11} + 2u_{10}u_{20}u_{30})\\
&+ 2\Pi_y y(-3Q_{0,9} - 18Q_{1,6}u_{30} + 6u_{01}u_{11} + 6u_{01}u_{20}u_{30} + 12u_{02}u_{10}\\
&+ 12u_{10}u_{11}u_{30} + u_{20}^3u_{30}) + 2\Pi_y(Q_{2,5} - 12uu_{11} - 12uu_{20}u_{30} - 4u_{01}u_{10}\\
&- 3u_{10}^2u_{30}) - 12\Pi x^3(u_{11}u_{31} + u_{21}^2) + 9\Pi x^2(-2u_{01}u_{31} + 2u_{02}u_{30}\\
&- 12u_{11}u_{21} + 2u_{12}u_{20} - u_{20}^2u_{31} - 2u_{20}u_{21}u_{30}) + 18\Pi x(-2Q_{0,7}u_{30}\\
&+ Q_{1,3}u_{31} - 2u_{01}u_{30}^2 - 2u_{10}u_{12} + 2u_{10}u_{20}u_{31} + 2u_{10}u_{21}u_{30} - 2u_{11}^2)\\
&+ 2\Pi y(-36Q_{0,7}u_{21} - 18Q_{1,6}u_{31} - 6u_{01}u_{12} + 6u_{01}u_{20}u_{31} - 30u_{01}u_{21}u_{30}\\
&- 12u_{02}u_{11} - 6u_{02}u_{20}u_{30} + 12u_{10}u_{11}u_{31} + 12u_{10}u_{21}^2 - 3u_{11}^2u_{30}\\
&+ 36u_{11}u_{20}u_{21} - 3u_{12}u_{20}^2 + u_{20}^3u_{31} + 3u_{20}^2u_{21}u_{30})\\
&+ 2\Pi(6Q_{0,9} + 18Q_{1,3}u_{21} + 18Q_{1,6}u_{30} + 12uu_{12} - 12uu_{20}u_{31}\\
&- 12uu_{21}u_{30} + 12u_{01}u_{11} + 6u_{01}u_{20}u_{30} - 12u_{02}u_{10} - 3u_{10}^2u_{31}\\
&- 6u_{10}u_{11}u_{30} - u_{20}^3u_{30}))/72
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(123) &= (-12\Pi_{yy}x^3u_{01}u_{30} + 9\Pi_{yy}x^2Q_{1,3}u_{30} + 18\Pi_{yy}xu_{10}^2u_{30}\\
&+ 2\Pi_{yy}yu_{30}(Q_{2,5} + 8u_{01}u_{10}) - 24\Pi_{yy}uu_{10}u_{30} + 24\Pi_{xy}x^3u_{01}u_{21}\\
&- 18\Pi_{xy}x^2Q_{1,3}u_{21} - 36\Pi_{xy}xu_{10}^2u_{21} + 4\Pi_{xy}yu_{21}(-Q_{2,5} - 8u_{01}u_{10})\\
&+ 48\Pi_{xy}uu_{10}u_{21} - 12\Pi_{xx}x^3u_{01}u_{12} + 9\Pi_{xx}x^2Q_{1,3}u_{12}\\
&+ 18\Pi_{xx}xu_{10}^2u_{12} + 2\Pi_{xx}yu_{12}(Q_{2,5} + 8u_{01}u_{10}) - 24\Pi_{xx}uu_{10}u_{12}
\end{aligned}
$$

$$
\begin{aligned}
&- 24\Pi_y x^3 u_{11} u_{21} + 18\Pi_y x^2 u_{21}(-2u_{01} - u_{20}^2)\\
&+ 36\Pi_y x u_{21}(Q_{1,3} + 2u_{10}u_{20})\\
&+ 4\Pi_y y u_{21}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&+ 12\Pi_y u_{21}(-4uu_{20} - u_{10}^2) + 12\Pi_x x^3 u_{11}u_{12}\\
&+ 9\Pi_x x^2 u_{12}(2u_{01} + u_{20}^2) + 18\Pi_x x u_{12}(-Q_{1,3} - 2u_{10}u_{20})\\
&+ 2\Pi_x y u_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&+ 6\Pi_x u_{12}(4uu_{20} + u_{10}^2) - 12\Pi x^3(u_{11}u_{22} + u_{12}u_{21})\\
&+ 9\Pi x^2(-2u_{01}u_{22} - 6u_{11}u_{12} - 2u_{12}u_{20}u_{30} - u_{20}^2 u_{22})\\
&+ 18\Pi x(Q_{1,3}u_{22} + 2u_{10}u_{12}u_{30} + 2u_{10}u_{20}u_{22})\\
&+ 2\Pi y(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22}\\
&+ 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22})\\
&+ 6\Pi(3Q_{1,3}u_{12} - 4uu_{12}u_{30} - 4uu_{20}u_{22} - u_{10}^2u_{22}))/72
\end{aligned}
$$

$$
\begin{aligned}
D_x\Pi(124) &= (\Pi_{yy}x^4 - 8\Pi_{yy}xyu_{10} - 8\Pi_{yy}y^2u_{01} + 16\Pi_{yy}yu - 4\Pi_y x^3 u_{30}\\
&+ 8\Pi_y xy(u_{11} + u_{20}u_{30}) + 8\Pi_y x u_{10} + 8\Pi_y y^2(u_{02} + u_{11}u_{30})\\
&- 8\Pi_y y u_{10}u_{30} - 16\Pi_y u - 4\Pi x^3 u_{31} - 24\Pi x^2 u_{21}\\
&+ 8\Pi xy(-u_{12} + u_{20}u_{31} + u_{21}u_{30}) + 8\Pi x(-2u_{11} - u_{20}u_{30})\\
&+ 8\Pi y^2(u_{11}u_{31} + u_{21}^2) + 8\Pi y(-2u_{02} - u_{10}u_{31} - u_{11}u_{30})\\
&+ 8\Pi(2u_{01} + u_{10}u_{30}))/8
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(124) &= (-\Pi_{yy}x^4u_{30} + 8\Pi_{yy}xyu_{10}u_{30} + 8\Pi_{yy}y^2u_{01}u_{30}\\
&- 16\Pi_{yy}yuu_{30} + 2\Pi_{xy}x^4u_{21} - 16\Pi_{xy}xyu_{10}u_{21} - 16\Pi_{xy}y^2u_{01}u_{21}\\
&+ 32\Pi_{xy}yuu_{21} - \Pi_{xx}x^4u_{12} + 8\Pi_{xx}xyu_{10}u_{12} + 8\Pi_{xx}y^2u_{01}u_{12}\\
&- 16\Pi_{xx}yuu_{12} - 8\Pi_y x^3u_{21} + 16\Pi_y xyu_{20}u_{21} + 16\Pi_y y^2u_{11}u_{21}\\
&- 16\Pi_y yu_{10}u_{21} + 4\Pi_x x^3u_{12} - 8\Pi_x xyu_{12}u_{20} - 8\Pi_x y^2u_{11}u_{12}\\
&+ 8\Pi_x yu_{10}u_{12} - 4\Pi x^3u_{22} - 12\Pi x^2u_{12} + 8\Pi xy(u_{12}u_{30} + u_{20}u_{22})\\
&+ 8\Pi y^2(u_{11}u_{22} + u_{12}u_{21}) - 8\Pi yu_{10}u_{22})/8
\end{aligned}
$$

$$
\begin{aligned}
D_x\Pi(125) &= (-3\Pi_{yy}x^4u_{01} + 3\Pi_{yy}x^3Q_{1,3} + 9\Pi_{yy}x^2u_{10}^2 + 2\Pi_{yy}xy(Q_{2,5} + 8u_{01}u_{10})\\
&- 24\Pi_{yy}xuu_{10} + 6\Pi_{yy}y^2(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 6\Pi_{yy}y(-2Q_{3,4} + Q_{1,3}u_{10} - 8uu_{01}) + 24\Pi_{yy}u^2 + 3\Pi_y x^4(u_{02} + u_{11}u_{30})\\
&+ 3\Pi_y x^3(-2Q_{0,7} + 2u_{11}u_{20} + u_{20}^2u_{30})\\
&+ 9\Pi_y x^2(-Q_{1,3}u_{30} - 2u_{10}u_{11} - 2u_{10}u_{20}u_{30})\\
&+ 2\Pi_y xy(3Q_{0,9} + 18Q_{1,6}u_{30} - 6u_{01}u_{11} - 6u_{01}u_{20}u_{30}\\
&- 12u_{02}u_{10} - 12u_{10}u_{11}u_{30} - u_{20}^3u_{30})\\
&+ 2\Pi_y x(-Q_{2,5} + 12uu_{11} + 12uu_{20}u_{30} + 4u_{01}u_{10} + 3u_{10}^2u_{30})\\
&+ 6\Pi_y y^2(4Q_{0,12} - 2Q_{0,7}u_{11} - 2Q_{0,7}u_{20}u_{30} + Q_{0,9}u_{30} - 4u_{01}u_{02}\\
&- 6u_{01}u_{11}u_{30} - 2u_{01}u_{20}u_{30}^2 + 2u_{11}^2u_{20} + u_{11}u_{20}^2u_{30})\\
&+ 2\Pi_y y(Q_{2,5}u_{30} - 6Q_{0,7}u_{10} + 3Q_{1,3}u_{11} + 3Q_{1,3}u_{20}u_{30} + 24uu_{02}\\
&+ 24uu_{11}u_{30} - 4u_{01}u_{10}u_{30} + 6u_{10}u_{11}u_{20} + 3u_{10}u_{20}^2u_{30})\\
&+ 6\Pi_y(2Q_{3,4} - Q_{1,3}u_{10} - 4uu_{10}u_{30}) + 3\Pi x^4(u_{11}u_{31} + u_{21}^2)\\
&+ 3\Pi x^3(2u_{01}u_{31} - 2u_{02}u_{30} + 12u_{11}u_{21} - 2u_{12}u_{20} + u_{20}^2u_{31} + 2u_{20}u_{21}u_{30})\\
&+ 9\Pi x^2(2Q_{0,7}u_{30} - Q_{1,3}u_{31} + 2u_{01}u_{30}^2 + 2u_{10}u_{12} - 2u_{10}u_{20}u_{31}\\
&- 2u_{10}u_{21}u_{30} + 2u_{11}^2) + 2\Pi xy(36Q_{0,7}u_{21} + 18Q_{1,6}u_{31} + 6u_{01}u_{12}
\end{aligned}
$$

$$
\begin{aligned}
&- 6u_{01}u_{20}u_{31} + 30u_{01}u_{21}u_{30} + 12u_{02}u_{11} + 6u_{02}u_{20}u_{30} - 12u_{10}u_{11}u_{31}\\
&- 12u_{10}u_{21}^2 + 3u_{11}^2u_{30} - 36u_{11}u_{20}u_{21} + 3u_{12}u_{20}^2 - u_{20}^3u_{31} - 3u_{20}^2u_{21}u_{30})\\
&+ 2\Pi x(-6Q_{0,9} - 18Q_{1,3}u_{21} - 18Q_{1,6}u_{30} - 12uu_{12} + 12uu_{20}u_{31}\\
&+ 12uu_{21}u_{30} - 12u_{01}u_{11} - 6u_{01}u_{20}u_{30} + 12u_{02}u_{10} + 3u_{10}^2u_{31}\\
&+ 6u_{10}u_{11}u_{30} + u_{20}^3u_{30}) + 6\Pi y^2(2Q_{0,7}u_{12} - 2Q_{0,7}u_{20}u_{31} - 2Q_{0,7}u_{21}u_{30}\\
&+ Q_{0,9}u_{31} - 4u_{01}u_{11}u_{31} + 2u_{01}u_{12}u_{30} - 2u_{01}u_{20}u_{30}u_{31} - 4u_{01}u_{21}^2\\
&- 2u_{01}u_{21}u_{30}^2 + 2u_{02}^2 + 2u_{02}u_{11}u_{30} - 6u_{11}^2u_{21} + u_{11}u_{20}^2u_{31})\\
&+ 2\Pi y(Q_{2,5}u_{31} - 24Q_{0,12} + 12Q_{0,7}u_{11} + 6Q_{0,7}u_{20}u_{30} - 3Q_{0,9}u_{30}\\
&- 3Q_{1,3}u_{12} + 3Q_{1,3}u_{20}u_{31} + 3Q_{1,3}u_{21}u_{30} + 24uu_{11}u_{31} + 24uu_{21}^2\\
&+ 2u_{01}u_{10}u_{31} + 12u_{01}u_{11}u_{30} + 6u_{01}u_{20}u_{30}^2 - 6u_{02}u_{10}u_{30} + 36u_{10}u_{11}u_{21}\\
&- 6u_{10}u_{12}u_{20} + 3u_{10}u_{20}^2u_{31} + 6u_{10}u_{20}u_{21}u_{30} - 12u_{11}^2u_{20} - 3u_{11}u_{20}^2u_{30})\\
&+ 2\Pi(-Q_{2,5}u_{30} - 6Q_{1,3}u_{11} - 3Q_{1,3}u_{20}u_{30} + 6Q_{1,8} - 24uu_{02}\\
&- 12uu_{10}u_{31} - 12uu_{11}u_{30} + 12u_{01}^2 + 22u_{01}u_{10}u_{30} + 6u_{01}u_{20}^2 - 18u_{10}^2u_{21}\\
&- 12u_{10}u_{11}u_{20} - 3u_{10}u_{20}^2u_{30}))/72
\end{aligned}
$$

$$
\begin{aligned}
D_y\Pi(125) = {}&(3\Pi_{yy}x^4u_{01}u_{30} - 3\Pi_{yy}x^3Q_{1,3}u_{30} - 9\Pi_{yy}x^2u_{10}^2u_{30}\\
&+ 2\Pi_{yy}xyu_{30}(-Q_{2,5} - 8u_{01}u_{10}) + 24\Pi_{yy}xuu_{10}u_{30}\\
&+ 6\Pi_{yy}y^2u_{30}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ 6\Pi_{yy}yu_{30}(2Q_{3,4} - Q_{1,3}u_{10} + 8uu_{01}) - 24\Pi_{yy}u^2u_{30}\\
&- 6\Pi_{xy}x^4u_{01}u_{21} + 6\Pi_{xy}x^3Q_{1,3}u_{21} + 18\Pi_{xy}x^2u_{10}^2u_{21}\\
&+ 4\Pi_{xy}xyu_{21}(Q_{2,5} + 8u_{01}u_{10}) - 48\Pi_{xy}xuu_{10}u_{21}\\
&+ 12\Pi_{xy}y^2u_{21}(2Q_{0,7}u_{10} - Q_{1,8} + 2u_{01}^2 - u_{01}u_{20}^2)\\
&+ 12\Pi_{xy}yu_{21}(-2Q_{3,4} + Q_{1,3}u_{10} - 8uu_{01}) + 48\Pi_{xy}u^2u_{21}\\
&+ 3\Pi_{xx}x^4u_{01}u_{12} - 3\Pi_{xx}x^3Q_{1,3}u_{12} - 9\Pi_{xx}x^2u_{10}^2u_{12}\\
&+ 2\Pi_{xx}xyu_{12}(-Q_{2,5} - 8u_{01}u_{10}) + 24\Pi_{xx}xuu_{10}u_{12}\\
&+ 6\Pi_{xx}y^2u_{12}(-2Q_{0,7}u_{10} + Q_{1,8} - 2u_{01}^2 + u_{01}u_{20}^2)\\
&+ 6\Pi_{xx}yu_{12}(2Q_{3,4} - Q_{1,3}u_{10} + 8uu_{01}) - 24\Pi_{xx}u^2u_{12} + 6\Pi_yx^4u_{11}u_{21}\\
&+ 6\Pi_yx^3u_{21}(2u_{01} + u_{20}^2) + 18\Pi_yx^2u_{21}(-Q_{1,3} - 2u_{10}u_{20})\\
&+ 4\Pi_yxyu_{21}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&+ 12\Pi_yxu_{21}(4uu_{20} + u_{10}^2)\\
&+ 12\Pi_yy^2u_{21}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 4\Pi_yyu_{21}(Q_{2,5} + 3Q_{1,3}u_{20} + 24uu_{11} + 2u_{01}u_{10} + 3u_{10}u_{20}^2)\\
&- 48\Pi_yuu_{10}u_{21} - 3\Pi_xx^4u_{11}u_{12} + 3\Pi_xx^3u_{12}(-2u_{01} - u_{20}^2)\\
&+ 9\Pi_xx^2u_{12}(Q_{1,3} + 2u_{10}u_{20})\\
&+ 2\Pi_xxyu_{12}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&+ 6\Pi_xxu_{12}(-4uu_{20} - u_{10}^2)\\
&+ 6\Pi_xy^2u_{12}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ 2\Pi_xyu_{12}(-Q_{2,5} - 3Q_{1,3}u_{20} - 24uu_{11} - 2u_{01}u_{10} - 3u_{10}u_{20}^2)\\
&+ 24\Pi_xuu_{10}u_{12} + 3\Pi x^4(u_{11}u_{22} + u_{12}u_{21})\\
&+ 3\Pi x^3(2u_{01}u_{22} + 6u_{11}u_{12} + 2u_{12}u_{20}u_{30} + u_{20}^2u_{22})\\
&+ 9\Pi x^2(-Q_{1,3}u_{22} - 2u_{10}u_{12}u_{30} - 2u_{10}u_{20}u_{22})\\
&+ 2\Pi xy(18Q_{0,7}u_{12} + 18Q_{1,6}u_{22} + 12u_{01}u_{12}u_{30} - 6u_{01}u_{20}u_{22}
\end{aligned}
$$

$$
\begin{aligned}
&- 12u_{10}u_{11}u_{22} - 12u_{10}u_{12}u_{21} - 18u_{11}u_{12}u_{20} - 3u_{12}u_{20}^2u_{30} - u_{20}^3u_{22})\\
&+ 6\Pi x(-3Q_{1,3}u_{12} + 4uu_{12}u_{30} + 4uu_{20}u_{22} + u_{10}^2u_{22})\\
&+ 6\Pi y^2(-2Q_{0,7}u_{12}u_{30} - 2Q_{0,7}u_{20}u_{22} + Q_{0,9}u_{22} - 4u_{01}u_{11}u_{22} - 4u_{01}u_{12}u_{21}\\
&- 2u_{01}u_{12}u_{30}^2 - 2u_{01}u_{20}u_{22}u_{30} - 3u_{11}^2u_{12} + u_{11}u_{20}^2u_{22})\\
&+ 2\Pi y(Q_{2,5}u_{22} + 3Q_{1,3}u_{12}u_{30} + 3Q_{1,3}u_{20}u_{22} + 24uu_{11}u_{22} + 24uu_{12}u_{21}\\
&+ 2u_{01}u_{10}u_{22} + 18u_{10}u_{11}u_{12} + 6u_{10}u_{12}u_{20}u_{30} + 3u_{10}u_{20}^2u_{22})\\
&+ 6\Pi u_{10}(-4uu_{22} - 3u_{10}u_{12}))/72
\end{aligned}
$$

According to the transformation of *nonlocal form-valued* variables, the associated x and y derivatives are given by

$$
\begin{aligned}
D_x\Omega(101) &= \Omega_{yy} - \Omega_y u_{40} - \Omega u_{41}\\
D_y\Omega(101) &= -\Omega_{yy}u_{30} + 2\Omega_{xy}u_{21} - \Omega_{xx}u_{12} - 2\Omega_y u_{31} + \Omega_x u_{22} - \Omega u_{32}\\
D_x\Omega(102) &= \Omega_{yy}u_{20} - \Omega_y(u_{20}u_{40} + u_{21} + u_{30}^2)\\
&\quad + \Omega(-u_{20}u_{41} - u_{21}u_{40} + u_{22} - 2u_{30}u_{31})\\
D_y\Omega(102) &= -\Omega_{yy}u_{20}u_{30} + 2\Omega_{xy}u_{20}u_{21} - \Omega_{xx}u_{12}u_{20} - 2\Omega_y(u_{20}u_{31} + u_{21}u_{30})\\
&\quad + \Omega_x(u_{12}u_{30} + u_{20}u_{22})\\
&\quad + \Omega(-u_{12}u_{40} - u_{20}u_{32} - 2u_{22}u_{30})\\
D_x\Omega(103) &= \Omega_{yy}u_{11} - \Omega_y(u_{11}u_{40} + u_{12} + u_{21}u_{30}) + \Omega(-u_{11}u_{41} - 3u_{21}u_{31})\\
D_y\Omega(103) &= -\Omega_{yy}u_{11}u_{30} + 2\Omega_{xy}u_{11}u_{21} - \Omega_{xx}u_{11}u_{12} - 2\Omega_y(u_{11}u_{31} + u_{21}^2)\\
&\quad + \Omega_x(u_{11}u_{22} + u_{12}u_{21})\\
&\quad + \Omega(-u_{11}u_{32} - u_{12}u_{31} - 2u_{21}u_{22})\\
D_x\Omega(104) &= \Omega_{yy}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + \Omega_y(18Q_{0,7}u_{30} + 18Q_{1,6}u_{40} - 6u_{01}u_{20}u_{40} - 6u_{01}u_{21}\\
&\quad + 12u_{01}u_{30}^2 - 6u_{02}u_{20} - 12u_{10}u_{11}u_{40} - 12u_{10}u_{12} - 12u_{10}u_{21}u_{30}\\
&\quad - 3u_{11}^2 - 18u_{11}u_{20}u_{30} - u_{20}^3u_{40} - 3u_{20}^2u_{21} - 3u_{20}^2u_{30}^2)\\
&\quad + \Omega(54Q_{0,7}u_{31} + 18Q_{1,6}u_{41} - 6u_{01}u_{20}u_{41} - 6u_{01}u_{21}u_{40}\\
&\quad + 6u_{01}u_{22} + 42u_{01}u_{30}u_{31} + 6u_{02}u_{20}u_{40} + 48u_{02}u_{21} + 6u_{02}u_{30}^2\\
&\quad - 12u_{10}u_{11}u_{41} - 36u_{10}u_{21}u_{31} + 3u_{11}^2u_{40} + 18u_{11}u_{12} - 54u_{11}u_{20}u_{31}\\
&\quad + 12u_{12}u_{20}u_{30} - u_{20}^3u_{41} - 3u_{20}^2u_{21}u_{40} + 3u_{20}^2u_{22} - 6u_{20}^2u_{30}u_{31}\\
&\quad - 48u_{20}u_{21}^2 - 6u_{20}u_{21}u_{30}^2)\\
D_y\Omega(104) &= \Omega_{yy}u_{30}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&\quad + 2\Omega_{xy}u_{21}(-18Q_{1,6} + 6u_{01}u_{20} + 12u_{10}u_{11} + u_{20}^3)\\
&\quad + \Omega_{xx}u_{12}(18Q_{1,6} - 6u_{01}u_{20} - 12u_{10}u_{11} - u_{20}^3)\\
&\quad + 2\Omega_y(18Q_{0,7}u_{21} + 18Q_{1,6}u_{31} - 6u_{01}u_{20}u_{31} + 12u_{01}u_{21}u_{30}\\
&\quad - 12u_{10}u_{11}u_{31} - 12u_{10}u_{21}^2 - 18u_{11}u_{20}u_{21} - u_{20}^3u_{31} - 3u_{20}^2u_{21}u_{30})\\
&\quad + \Omega_x(-18Q_{0,7}u_{12} - 18Q_{1,6}u_{22} - 12u_{01}u_{12}u_{30} + 6u_{01}u_{20}u_{22}\\
&\quad + 12u_{10}u_{11}u_{22} + 12u_{10}u_{12}u_{21} + 18u_{11}u_{12}u_{20} + 3u_{12}u_{20}^2u_{30} + u_{20}^3u_{22})\\
&\quad + \Omega(36Q_{0,7}u_{22} + 18Q_{1,6}u_{32} - 6u_{01}u_{12}u_{40} - 6u_{01}u_{20}u_{32} + 24u_{01}u_{22}u_{30}\\
&\quad + 18u_{02}u_{12} - 12u_{10}u_{11}u_{32}\\
&\quad - 12u_{10}u_{12}u_{31} - 24u_{10}u_{21}u_{22} - 6u_{11}u_{12}u_{30} - 36u_{11}u_{20}u_{22}\\
&\quad - 3u_{12}u_{20}^2u_{40} - 30u_{12}u_{20}u_{21} - 6u_{12}u_{20}u_{30}^2 - u_{20}^3u_{32} - 6u_{20}^2u_{22}u_{30})\\
D_x\Omega(105) &= \Omega_{yy}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)
\end{aligned}
$$

$$
\begin{aligned}
&+ \Omega_y(-2Q_{0,7}u_{20}u_{40} - 2Q_{0,7}u_{21} - 2Q_{0,7}u_{30}^2 + Q_{0,9}u_{40} - 4u_{01}u_{11}u_{40}\\
&- 4u_{01}u_{12} - 2u_{01}u_{20}u_{30}u_{40} - 6u_{01}u_{21}u_{30} - 2u_{01}u_{30}^3 - 2u_{02}u_{11}\\
&- 3u_{11}^2u_{30} + u_{11}u_{20}^2u_{40}) + \Omega(-2Q_{0,7}u_{20}u_{41} - 2Q_{0,7}u_{21}u_{40} + 2Q_{0,7}u_{22}\\
&- 4Q_{0,7}u_{30}u_{31} + Q_{0,9}u_{41} - 4u_{01}u_{11}u_{41} - 2u_{01}u_{20}u_{30}u_{41}\\
&- 2u_{01}u_{21}u_{30}u_{40} - 12u_{01}u_{21}u_{31} + 2u_{01}u_{22}u_{30} - 4u_{01}u_{30}^2u_{31}\\
&+ 2u_{02}u_{11}u_{40} + 6u_{02}u_{12} - 9u_{11}^2u_{31} + 4u_{11}u_{12}u_{30} + u_{11}u_{20}^2u_{41}\\
&- 16u_{11}u_{21}^2 - 2u_{11}u_{21}u_{30}^2)
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(105) = {}& \Omega_{yy}u_{30}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 2\Omega_{xy}u_{21}(2Q_{0,7}u_{20} - Q_{0,9} + 4u_{01}u_{11} + 2u_{01}u_{20}u_{30} - u_{11}u_{20}^2)\\
&+ \Omega_{xx}u_{12}(-2Q_{0,7}u_{20} + Q_{0,9} - 4u_{01}u_{11} - 2u_{01}u_{20}u_{30} + u_{11}u_{20}^2)\\
&+ 2\Omega_y(-2Q_{0,7}u_{20}u_{31} - 2Q_{0,7}u_{21}u_{30} + Q_{0,9}u_{31} - 4u_{01}u_{11}u_{31}\\
&- 2u_{01}u_{20}u_{30}u_{31} - 4u_{01}u_{21}^2 - 2u_{01}u_{21}u_{30}^2 - 3u_{11}^2u_{21} + u_{11}u_{20}^2u_{31})\\
&+ \Omega_x(2Q_{0,7}u_{12}u_{30} + 2Q_{0,7}u_{20}u_{22} - Q_{0,9}u_{22} + 4u_{01}u_{11}u_{22} + 4u_{01}u_{12}u_{21}\\
&+ 2u_{01}u_{12}u_{30}^2 + 2u_{01}u_{20}u_{22}u_{30} + 3u_{11}^2u_{12} - u_{11}u_{20}^2u_{22})\\
&+ \Omega(-2Q_{0,7}u_{12}u_{40} - 2Q_{0,7}u_{20}u_{32} - 4Q_{0,7}u_{22}u_{30} + Q_{0,9}u_{32}\\
&- 4u_{01}u_{11}u_{32} - 2u_{01}u_{12}u_{30}u_{40} - 4u_{01}u_{12}u_{31} - 2u_{01}u_{20}u_{30}u_{32}\\
&- 8u_{01}u_{21}u_{22} - 4u_{01}u_{22}u_{30}^2 - 2u_{02}u_{12}u_{30} - 6u_{11}^2u_{22}\\
&- 10u_{11}u_{12}u_{21} - 2u_{11}u_{12}u_{30}^2 + u_{11}u_{20}^2u_{32})
\end{aligned}
$$

$$
\begin{aligned}
D_x\Omega(106) = {}& \Omega_{yy}(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} + 360Q_{0,7}u_{10}u_{20}\\
&+ 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} + 600u_{01}^2u_{20}\\
&- 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 + 270u_{02}u_{10}^2\\
&+ 690u_{10}u_{11}u_{20}^2 + u_{20}^5) + \Omega_y(90Q_{2,10}u_{40} - 40Q_{2,5}u_{11}u_{40} - 40Q_{2,5}u_{12}\\
&- 40Q_{2,5}u_{21}u_{30} + 240Q_{0,12}u_{20} - 180Q_{0,7}^2 - 270Q_{0,7}Q_{1,3}u_{40}\\
&- 360Q_{0,7}u_{01}u_{30} - 360Q_{0,7}u_{10}u_{20}u_{40} - 360Q_{0,7}u_{10}u_{21} - 360Q_{0,7}u_{10}u_{30}^2\\
&- 60Q_{0,9}u_{10}u_{40} + 60Q_{0,9}u_{11} - 180Q_{1,11}u_{30} + 360Q_{1,6}u_{01}u_{40}\\
&+ 360Q_{1,6}u_{02} + 1080Q_{1,6}u_{11}u_{30} + 180Q_{1,6}u_{20}^2u_{40} + 360Q_{1,6}u_{20}u_{21}\\
&+ 360Q_{1,6}u_{20}u_{30}^2 + 60Q_{1,8}u_{20}u_{40} + 60Q_{1,8}u_{21} + 60Q_{1,8}u_{30}^2\\
&- 600u_{01}^2u_{20}u_{40} - 60u_{01}^2u_{21} - 60u_{01}^2u_{30}^2 - 120u_{01}u_{02}u_{20}\\
&+ 220u_{01}u_{10}u_{11}u_{40} - 320u_{01}u_{10}u_{12} + 1380u_{01}u_{10}u_{20}u_{30}u_{40}\\
&- 560u_{01}u_{10}u_{21}u_{30} - 240u_{01}u_{10}u_{30}^3 - 60u_{01}u_{11}^2 - 360u_{01}u_{11}u_{20}u_{30}\\
&+ 940u_{01}u_{20}^3u_{40} + 180u_{01}u_{20}^2u_{30}^2 - 270u_{02}u_{10}^2u_{40} - 240u_{02}u_{10}u_{11}\\
&- 20u_{02}u_{20}^3 - 360u_{10}u_{11}^2u_{30} - 690u_{10}u_{11}u_{20}^2u_{40} + 150u_{11}^2u_{20}^2\\
&- 60u_{11}u_{20}^3u_{30} - u_{20}^5u_{40} - 5u_{20}^4u_{21} - 5u_{20}^4u_{30}^2)\\
&+ \Omega(90Q_{2,10}u_{41} - 40Q_{2,5}u_{11}u_{41} - 120Q_{2,5}u_{21}u_{31} - 240Q_{0,12}u_{20}u_{40}\\
&- 1920Q_{0,12}u_{21} - 240Q_{0,12}u_{30}^2 + 180Q_{0,7}^2u_{40} - 270Q_{0,7}Q_{1,3}u_{41}\\
&+ 360Q_{0,7}u_{01}u_{30}u_{40} - 360Q_{0,7}u_{10}u_{20}u_{41} - 360Q_{0,7}u_{10}u_{21}u_{40}\\
&+ 360Q_{0,7}u_{10}u_{22} - 720Q_{0,7}u_{10}u_{30}u_{31} + 2880Q_{0,7}u_{11}u_{21}\\
&+ 360Q_{0,7}u_{11}u_{30}^2 - 60Q_{0,9}u_{10}u_{41} - 60Q_{0,9}u_{11}u_{40} - 180Q_{0,9}u_{12}\\
&- 540Q_{1,11}u_{31} + 360Q_{1,6}u_{01}u_{41} - 360Q_{1,6}u_{02}u_{40} + 3240Q_{1,6}u_{11}u_{31}\\
&- 720Q_{1,6}u_{12}u_{30} + 180Q_{1,6}u_{20}^2u_{41} + 360Q_{1,6}u_{20}u_{21}u_{40} - 360Q_{1,6}u_{20}u_{22}\\
&+ 720Q_{1,6}u_{20}u_{30}u_{31} + 2880Q_{1,6}u_{21}^2 + 360Q_{1,6}u_{21}u_{30}^2 + 60Q_{1,8}u_{20}u_{41}
\end{aligned}
$$

$$
\begin{aligned}
&+ 60Q_{1,8}u_{21}u_{40} - 60Q_{1,8}u_{22} + 120Q_{1,8}u_{30}u_{31} - 600u_{01}^2u_{20}u_{41} \\
&- 60u_{01}^2u_{21}u_{40} + 60u_{01}^2u_{22} + 180u_{01}^2u_{30}^2u_{40} + 420u_{01}^2u_{30}u_{31} \\
&+ 120u_{01}u_{02}u_{20}u_{40} + 960u_{01}u_{02}u_{21} + 120u_{01}u_{02}u_{30}^2 + 220u_{01}u_{10}u_{11}u_{41} \\
&+ 1380u_{01}u_{10}u_{20}u_{30}u_{41} - 240u_{01}u_{10}u_{21}u_{30}u_{40} \\
&- 960u_{01}u_{10}u_{21}u_{31} + 240u_{01}u_{10}u_{22}u_{30} - 480u_{01}u_{10}u_{30}^2u_{31} \\
&+ 60u_{01}u_{11}^2u_{40} + 360u_{01}u_{11}u_{12} - 1080u_{01}u_{11}u_{20}u_{31} + 2880u_{01}u_{11}u_{21}u_{30} \\
&+ 360u_{01}u_{11}u_{30}^3 + 240u_{01}u_{12}u_{20}u_{30} + 940u_{01}u_{20}^3u_{41} \\
&+ 540u_{01}u_{20}^2u_{30}u_{31} - 960u_{01}u_{20}u_{21}^2 - 120u_{01}u_{20}u_{21}u_{30}^2 - 270u_{02}u_{10}^2u_{41} \\
&+ 240u_{02}u_{10}u_{11}u_{40} + 720u_{02}u_{10}u_{12} + 20u_{02}u_{20}^3u_{40} + 480u_{02}u_{20}^2u_{21} \\
&+ 60u_{02}u_{20}^2u_{30}^2 - 1080u_{10}u_{11}^2u_{31} + 480u_{10}u_{11}u_{12}u_{30} - 690u_{10}u_{11}u_{20}^2u_{41} \\
&- 1920u_{10}u_{11}u_{21}^2 - 240u_{10}u_{11}u_{21}u_{30}^2 - 150u_{11}^2u_{20}^2u_{40} \\
&- 2400u_{11}^2u_{20}u_{21} - 300u_{11}^2u_{20}u_{30}^2 - 180u_{11}u_{20}^3u_{31} + 40u_{12}u_{20}^3u_{30} - u_{20}^5u_{41} \\
&- 5u_{20}^4u_{21}u_{40} + 5u_{20}^4u_{22} - 10u_{20}^4u_{30}u_{31} - 160u_{20}^3u_{21}^2 - 20u_{20}^3u_{21}u_{30}^2)
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(106) &= \Omega_{yy}u_{30}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20} \\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2 + 60Q_{1,8}u_{20} - 600u_{01}^2u_{20} \\
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2 \\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 2\Omega_{xy}u_{21}(-90Q_{2,10} + 40Q_{2,5}u_{11} + 270Q_{0,7}Q_{1,3} \\
&+ 360Q_{0,7}u_{10}u_{20} + 60Q_{0,9}u_{10} - 360Q_{1,6}u_{01} - 180Q_{1,6}u_{20}^2 - 60Q_{1,8}u_{20} \\
&+ 600u_{01}^2u_{20} - 220u_{01}u_{10}u_{11} - 1380u_{01}u_{10}u_{20}u_{30} - 940u_{01}u_{20}^3 \\
&+ 270u_{02}u_{10}^2 + 690u_{10}u_{11}u_{20}^2 + u_{20}^5) \\
&+ \Omega_{xx}u_{12}(90Q_{2,10} - 40Q_{2,5}u_{11} - 270Q_{0,7}Q_{1,3} - 360Q_{0,7}u_{10}u_{20} \\
&- 60Q_{0,9}u_{10} + 360Q_{1,6}u_{01} + 180Q_{1,6}u_{20}^2 + 60Q_{1,8}u_{20} - 600u_{01}^2u_{20} \\
&+ 220u_{01}u_{10}u_{11} + 1380u_{01}u_{10}u_{20}u_{30} + 940u_{01}u_{20}^3 - 270u_{02}u_{10}^2 \\
&- 690u_{10}u_{11}u_{20}^2 - u_{20}^5) + 2\Omega_y(90Q_{2,10}u_{31} - 40Q_{2,5}u_{11}u_{31} - 40Q_{2,5}u_{21}^2 \\
&- 270Q_{0,7}Q_{1,3}u_{31} - 360Q_{0,7}u_{10}u_{20}u_{31} - 360Q_{0,7}u_{10}u_{21}u_{30} \\
&- 60Q_{0,9}u_{10}u_{31} - 180Q_{1,11}u_{21} + 360Q_{1,6}u_{01}u_{31} + 1080Q_{1,6}u_{11}u_{21} \\
&+ 180Q_{1,6}u_{20}^2u_{31} + 360Q_{1,6}u_{20}u_{21}u_{30} + 60Q_{1,8}u_{20}u_{31} + 60Q_{1,8}u_{21}u_{30} \\
&- 600u_{01}^2u_{20}u_{31} + 120u_{01}^2u_{21}u_{30} + 220u_{01}u_{10}u_{11}u_{31} \\
&+ 1380u_{01}u_{10}u_{20}u_{30}u_{31} - 320u_{01}u_{10}u_{21}^2 - 240u_{01}u_{10}u_{21}u_{30}^2 \\
&- 360u_{01}u_{11}u_{20}u_{21} + 940u_{01}u_{20}^3u_{31} + 180u_{01}u_{20}^2u_{21}u_{30} - 270u_{02}u_{10}^2u_{31} \\
&- 360u_{10}u_{11}^2u_{21} - 690u_{10}u_{11}u_{20}^2u_{31} - 60u_{11}u_{20}^3u_{21} - u_{20}^5u_{31} - 5u_{20}^4u_{21}u_{30}) \\
&+ \Omega_x(-90Q_{2,10}u_{22} + 40Q_{2,5}u_{11}u_{22} + 40Q_{2,5}u_{12}u_{21} + 270Q_{0,7}Q_{1,3}u_{22} \\
&+ 360Q_{0,7}u_{10}u_{12}u_{30} + 360Q_{0,7}u_{10}u_{20}u_{22} + 60Q_{0,9}u_{10}u_{22} \\
&+ 180Q_{1,11}u_{12} - 360Q_{1,6}u_{01}u_{22} - 1080Q_{1,6}u_{11}u_{12} \\
&- 360Q_{1,6}u_{12}u_{20}u_{30} - 180Q_{1,6}u_{20}^2u_{22} - 60Q_{1,8}u_{12}u_{30} - 60Q_{1,8}u_{20}u_{22} \\
&- 120u_{01}^2u_{12}u_{30} + 600u_{01}^2u_{20}u_{22} - 220u_{01}u_{10}u_{11}u_{22} + 320u_{01}u_{10}u_{12}u_{21} \\
&+ 240u_{01}u_{10}u_{12}u_{30}^2 - 1380u_{01}u_{10}u_{20}u_{22}u_{30} + 360u_{01}u_{11}u_{12}u_{20} \\
&- 180u_{01}u_{12}u_{20}^2u_{30} - 940u_{01}u_{20}^3u_{22} + 270u_{02}u_{10}^2u_{22} + 360u_{10}u_{11}^2u_{12} \\
&+ 690u_{10}u_{11}u_{20}^2u_{22} + 60u_{11}u_{12}u_{20}^3 + 5u_{12}u_{20}^4u_{30} + u_{20}^5u_{22}) \\
&+ \Omega(90Q_{2,10}u_{32} - 40Q_{2,5}u_{11}u_{32} - 40Q_{2,5}u_{12}u_{31} - 80Q_{2,5}u_{21}u_{22} \\
&- 720Q_{0,12}u_{12} - 270Q_{0,7}Q_{1,3}u_{32} - 360Q_{0,7}u_{10}u_{12}u_{40} - 360Q_{0,7}u_{10}u_{20}u_{32}
\end{aligned}
$$

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\begin{aligned}
&- 720Q_{0,7}u_{10}u_{22}u_{30} + 1080Q_{0,7}u_{11}u_{12} - 60Q_{0,9}u_{10}u_{32}\\
&+ 60Q_{0,9}u_{12}u_{30} - 360Q_{1,11}u_{22} + 360Q_{1,6}u_{01}u_{32} + 2160Q_{1,6}u_{11}u_{22}\\
&+ 360Q_{1,6}u_{12}u_{20}u_{40} + 1800Q_{1,6}u_{12}u_{21} + 360Q_{1,6}u_{12}u_{30}^2 + 180Q_{1,6}u_{20}^2u_{32}\\
&+ 720Q_{1,6}u_{20}u_{22}u_{30} + 60Q_{1,8}u_{12}u_{40} + 60Q_{1,8}u_{20}u_{32} + 120Q_{1,8}u_{22}u_{30}\\
&- 60u_{01}^2u_{12}u_{40} - 600u_{01}^2u_{20}u_{32} + 240u_{01}^2u_{22}u_{30} + 360u_{01}u_{02}u_{12}\\
&+ 220u_{01}u_{10}u_{11}u_{32} - 240u_{01}u_{10}u_{12}u_{30}u_{40} - 320u_{01}u_{10}u_{12}u_{31}\\
&+ 1380u_{01}u_{10}u_{20}u_{30}u_{32} - 640u_{01}u_{10}u_{21}u_{22} - 480u_{01}u_{10}u_{22}u_{30}^2\\
&+ 960u_{01}u_{11}u_{12}u_{30} - 720u_{01}u_{11}u_{20}u_{22} - 600u_{01}u_{12}u_{20}u_{21}\\
&- 120u_{01}u_{12}u_{20}u_{30}^2 + 940u_{01}u_{20}^3u_{32} + 360u_{01}u_{20}^2u_{22}u_{30} - 270u_{02}u_{10}^2u_{32}\\
&- 240u_{02}u_{10}u_{12}u_{30} + 180u_{02}u_{12}u_{20}^2 - 720u_{10}u_{11}^2u_{22} - 1200u_{10}u_{11}u_{12}u_{21}\\
&- 240u_{10}u_{11}u_{12}u_{30}^2 - 690u_{10}u_{11}u_{20}^2u_{32} - 900u_{11}^2u_{12}u_{20} - 120u_{11}u_{20}^3u_{22}\\
&- 5u_{12}u_{20}^4u_{40} - 100u_{12}u_{20}^3u_{21} - 20u_{12}u_{20}^3u_{30}^2 - u_{20}^5u_{32} - 10u_{20}^4u_{22}u_{30})\\
D_x\Omega(107) &= \Omega_{yy}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3\\
&- 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11} + 195u_{01}^2u_{11}\\
&- 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30} - 20u_{01}u_{10}u_{20}u_{21}\\
&- 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30} - 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22}\\
&+ 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4) + \Omega_y(-288Q_{0,12}u_{10}u_{40}\\
&+ 288Q_{0,12}u_{11} + 216Q_{0,7}^2u_{30} - 144Q_{0,7}u_{01}u_{20}u_{40} - 144Q_{0,7}u_{01}u_{21}\\
&+ 288Q_{0,7}u_{01}u_{30}^2 - 144Q_{0,7}u_{02}u_{20} - 288Q_{0,7}u_{10}u_{11}u_{40} - 288Q_{0,7}u_{10}u_{12}\\
&- 288Q_{0,7}u_{10}u_{21}u_{30} - 72Q_{0,7}u_{11}^2 - 432Q_{0,7}u_{11}u_{20}u_{30} - 24Q_{0,7}u_{20}^3u_{40}\\
&- 72Q_{0,7}u_{20}^2u_{21} - 72Q_{0,7}u_{20}^2u_{30}^2 + 72Q_{0,9}u_{01}u_{40} + 72Q_{0,9}u_{02}\\
&+ 216Q_{0,9}u_{11}u_{30} + 36Q_{0,9}u_{20}^2u_{40} + 72Q_{0,9}u_{20}u_{21} + 72Q_{0,9}u_{20}u_{30}^2\\
&+ 72Q_{1,11}u_{20}u_{40} + 72Q_{1,11}u_{21} + 72Q_{1,11}u_{30}^2 + Q_{1,13}u_{40} + 144Q_{1,8}u_{11}u_{40}\\
&+ 144Q_{1,8}u_{12} + 144Q_{1,8}u_{21}u_{30} - 195u_{01}^2u_{11}u_{40} - 288u_{01}^2u_{12} - 504u_{01}^2u_{21}u_{30}\\
&+ 156u_{01}u_{02}u_{10}u_{40} - 288u_{01}u_{02}u_{11} - 144u_{01}u_{02}u_{20}u_{30} - 10u_{01}u_{10}^2u_{31}u_{40}\\
&- 20u_{01}u_{10}u_{11}u_{30}u_{40} + 20u_{01}u_{10}u_{20}u_{21}u_{40} - 504u_{01}u_{11}^2u_{30}\\
&+ 82u_{01}u_{11}u_{20}^2u_{40} - 432u_{01}u_{11}u_{20}u_{30}^2 + 144u_{01}u_{12}u_{20}^2 - 96u_{01}u_{20}^3u_{30}u_{40}\\
&- 144u_{01}u_{20}^2u_{30}^3 - 12u_{02}u_{10}^2u_{30}u_{40} + 12u_{02}u_{10}u_{20}^2u_{40} + 72u_{02}u_{11}u_{20}^2\\
&+ 2u_{10}^3u_{22}u_{40} - 6u_{10}^2u_{12}u_{20}u_{40} + 20u_{10}u_{11}^2u_{20}u_{40}\\
&+ 144u_{11}^3u_{20} + 216u_{11}^2u_{20}^2u_{30} + 42u_{11}u_{20}^4u_{40} + 72u_{11}u_{20}^3u_{21} + 72u_{11}u_{20}^3u_{30}^2)\\
&+ \Omega(-288Q_{0,12}u_{10}u_{41} - 288Q_{0,12}u_{11}u_{40} - 864Q_{0,12}u_{12} + 648Q_{0,7}^2u_{31}\\
&- 144Q_{0,7}u_{01}u_{20}u_{41} - 144Q_{0,7}u_{01}u_{21}u_{40} + 144Q_{0,7}u_{01}u_{22}\\
&+ 1008Q_{0,7}u_{01}u_{30}u_{31} + 144Q_{0,7}u_{02}u_{20}u_{40} + 1152Q_{0,7}u_{02}u_{21}\\
&+ 144Q_{0,7}u_{02}u_{30}^2 - 288Q_{0,7}u_{10}u_{11}u_{41} - 864Q_{0,7}u_{10}u_{21}u_{31}\\
&+ 72Q_{0,7}u_{11}^2u_{40} + 432Q_{0,7}u_{11}u_{12} - 1296Q_{0,7}u_{11}u_{20}u_{31} + 288Q_{0,7}u_{12}u_{20}u_{30}\\
&- 24Q_{0,7}u_{20}^3u_{41} - 72Q_{0,7}u_{20}^2u_{21}u_{40} + 72Q_{0,7}u_{20}^2u_{22}\\
&- 144Q_{0,7}u_{20}^2u_{30}u_{31} - 1152Q_{0,7}u_{20}u_{21}^2 - 144Q_{0,7}u_{20}u_{21}u_{30}^2 + 72Q_{0,9}u_{01}u_{41}\\
&- 72Q_{0,9}u_{02}u_{40} + 648Q_{0,9}u_{11}u_{31} - 144Q_{0,9}u_{12}u_{30} + 36Q_{0,9}u_{20}^2u_{41}\\
&+ 72Q_{0,9}u_{20}u_{21}u_{40} - 72Q_{0,9}u_{20}u_{22} + 144Q_{0,9}u_{20}u_{30}u_{31} + 576Q_{0,9}u_{21}^2\\
&+ 72Q_{0,9}u_{21}u_{30}^2 + 72Q_{1,11}u_{20}u_{41} + 72Q_{1,11}u_{21}u_{40} - 72Q_{1,11}u_{22}\\
&+ 144Q_{1,11}u_{30}u_{31} + Q_{1,13}u_{41} + 144Q_{1,8}u_{11}u_{41} + 432Q_{1,8}u_{21}u_{31}
\end{aligned}
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\begin{aligned}
&- 195u_{01}^2u_{11}u_{41} - 216u_{01}^2u_{21}u_{30}u_{40} - 864u_{01}^2u_{21}u_{31} + 216u_{01}^2u_{22}u_{30}\\
&+ 216u_{01}^2u_{30}^2u_{31} + 156u_{01}u_{02}u_{10}u_{41} + 288u_{01}u_{02}u_{11}u_{40}\\
&+ 864u_{01}u_{02}u_{12} + 144u_{01}u_{02}u_{20}u_{30}u_{40} + 1152u_{01}u_{02}u_{21}u_{30}\\
&+ 144u_{01}u_{02}u_{30}^3 - 10u_{01}u_{10}^2u_{31}u_{41} - 20u_{01}u_{10}u_{11}u_{30}u_{41}\\
&+ 20u_{01}u_{10}u_{20}u_{21}u_{41} + 72u_{01}u_{11}^2u_{30}u_{40} - 1296u_{01}u_{11}^2u_{31}\\
&+ 1008u_{01}u_{11}u_{12}u_{30} + 82u_{01}u_{11}u_{20}^2u_{41} - 1296u_{01}u_{11}u_{20}u_{30}u_{31}\\
&- 2304u_{01}u_{11}u_{21}^2 - 288u_{01}u_{11}u_{21}u_{30}^2 + 288u_{01}u_{12}u_{20}u_{30}^2\\
&- 96u_{01}u_{20}^3u_{30}u_{41} - 144u_{01}u_{20}^2u_{21}u_{30}u_{40} + 432u_{01}u_{20}^2u_{21}u_{31}\\
&+ 144u_{01}u_{20}^2u_{22}u_{30} - 288u_{01}u_{20}^2u_{30}^2u_{31} - 1152u_{01}u_{20}u_{21}^2u_{30}\\
&- 144u_{01}u_{20}u_{21}u_{30}^3 - 12u_{02}u_{10}^2u_{30}u_{41}\\
&+ 12u_{02}u_{10}u_{20}^2u_{41} - 72u_{02}u_{11}u_{20}^2u_{40} + 2u_{10}^3u_{22}u_{41} - 6u_{10}^2u_{12}u_{20}u_{41}\\
&+ 20u_{10}u_{11}^2u_{20}u_{41} - 144u_{11}^3u_{20}u_{40} - 576u_{11}^3u_{21} - 72u_{11}^3u_{30}^2\\
&- 432u_{11}^2u_{12}u_{20} + 648u_{11}^2u_{20}^2u_{31} - 144u_{11}u_{12}u_{20}^2u_{30}\\
&+ 42u_{11}u_{20}^4u_{41} + 72u_{11}u_{20}^3u_{21}u_{40} - 72u_{11}u_{20}^3u_{22}\\
&+ 144u_{11}u_{20}^3u_{30}u_{31} + 576u_{11}u_{20}^2u_{21}^2 + 72u_{11}u_{20}^2u_{21}u_{30}^2)
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(107) = {}&\Omega_{yy}u_{30}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11} - 24Q_{0,7}u_{20}^3\\
&+ 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13} + 144Q_{1,8}u_{11} - 195u_{01}^2u_{11}\\
&+ 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31} - 20u_{01}u_{10}u_{11}u_{30} + 20u_{01}u_{10}u_{20}u_{21}\\
&+ 82u_{01}u_{11}u_{20}^2 - 96u_{01}u_{20}^3u_{30} - 12u_{02}u_{10}^2u_{30}\\
&+ 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20} + 20u_{10}u_{11}^2u_{20} + 42u_{11}u_{20}^4)\\
&+ 2\Omega_{xy}u_{21}(288Q_{0,12}u_{10} + 144Q_{0,7}u_{01}u_{20} + 288Q_{0,7}u_{10}u_{11} + 24Q_{0,7}u_{20}^3\\
&- 72Q_{0,9}u_{01} - 36Q_{0,9}u_{20}^2 - 72Q_{1,11}u_{20} - Q_{1,13} - 144Q_{1,8}u_{11}\\
&+ 195u_{01}^2u_{11} - 156u_{01}u_{02}u_{10} + 10u_{01}u_{10}^2u_{31} + 20u_{01}u_{10}u_{11}u_{30}\\
&- 20u_{01}u_{10}u_{20}u_{21} - 82u_{01}u_{11}u_{20}^2 + 96u_{01}u_{20}^3u_{30} + 12u_{02}u_{10}^2u_{30}\\
&- 12u_{02}u_{10}u_{20}^2 - 2u_{10}^3u_{22} + 6u_{10}^2u_{12}u_{20} - 20u_{10}u_{11}^2u_{20} - 42u_{11}u_{20}^4)\\
&+ \Omega_{xx}u_{12}(-288Q_{0,12}u_{10} - 144Q_{0,7}u_{01}u_{20} - 288Q_{0,7}u_{10}u_{11} - 24Q_{0,7}u_{20}^3\\
&+ 72Q_{0,9}u_{01} + 36Q_{0,9}u_{20}^2 + 72Q_{1,11}u_{20} + Q_{1,13} + 144Q_{1,8}u_{11} - 195u_{01}^2u_{11}\\
&+ 156u_{01}u_{02}u_{10} - 10u_{01}u_{10}^2u_{31} - 20u_{01}u_{10}u_{11}u_{30}\\
&+ 20u_{01}u_{10}u_{20}u_{21} + 82u_{01}u_{11}u_{20}^2 - 96u_{01}u_{20}^3u_{30} - 12u_{02}u_{10}^2u_{30}\\
&+ 12u_{02}u_{10}u_{20}^2 + 2u_{10}^3u_{22} - 6u_{10}^2u_{12}u_{20} + 20u_{10}u_{11}^2u_{20} + 42u_{11}u_{20}^4)\\
&+ 2\Omega_y(-288Q_{0,12}u_{10}u_{31} + 216Q_{0,7}^2u_{21} - 144Q_{0,7}u_{01}u_{20}u_{31}\\
&+ 288Q_{0,7}u_{01}u_{21}u_{30} - 288Q_{0,7}u_{10}u_{11}u_{31} - 288Q_{0,7}u_{10}u_{21}^2\\
&- 432Q_{0,7}u_{11}u_{20}u_{21} - 24Q_{0,7}u_{20}^3u_{31} - 72Q_{0,7}u_{20}^2u_{21}u_{30} + 72Q_{0,9}u_{01}u_{31}\\
&+ 216Q_{0,9}u_{11}u_{21} + 36Q_{0,9}u_{20}^2u_{31} + 72Q_{0,9}u_{20}u_{21}u_{30} + 72Q_{1,11}u_{20}u_{31}\\
&+ 72Q_{1,11}u_{21}u_{30} + Q_{1,13}u_{31} + 144Q_{1,8}u_{11}u_{31} + 144Q_{1,8}u_{21}^2\\
&- 195u_{01}^2u_{11}u_{31} - 288u_{01}^2u_{21}^2 + 156u_{01}u_{02}u_{10}u_{31} - 10u_{01}u_{10}^2u_{31}^2\\
&- 20u_{01}u_{10}u_{11}u_{30}u_{31} + 20u_{01}u_{10}u_{20}u_{21}u_{31} - 432u_{01}u_{11}^2u_{21}\\
&+ 82u_{01}u_{11}u_{20}^2u_{31} - 432u_{01}u_{11}u_{20}u_{21}u_{30} - 96u_{01}u_{20}^3u_{30}u_{31}\\
&+ 144u_{01}u_{20}^2u_{21}^2 - 144u_{01}u_{20}^2u_{21}u_{30}^2 - 12u_{02}u_{10}^2u_{30}u_{31}\\
&+ 12u_{02}u_{10}u_{20}^2u_{31} + 2u_{10}^3u_{22}u_{31} - 6u_{10}^2u_{12}u_{20}u_{31} + 20u_{10}u_{11}^2u_{20}u_{31}
\end{aligned}
$$

$$
\begin{aligned}
&+ 216u_{11}^2u_{20}^2u_{21} + 42u_{11}u_{20}^4u_{31} + 72u_{11}u_{20}^3u_{21}u_{30})\\
&+ \Omega_x(288Q_{0,12}u_{10}u_{22} - 216Q_{0,7}^2u_{12} - 288Q_{0,7}u_{01}u_{12}u_{30}\\
&+ 144Q_{0,7}u_{01}u_{20}u_{22} + 288Q_{0,7}u_{10}u_{11}u_{22} + 288Q_{0,7}u_{10}u_{12}u_{21}\\
&+ 432Q_{0,7}u_{11}u_{12}u_{20} + 72Q_{0,7}u_{12}u_{20}^2u_{30} + 24Q_{0,7}u_{20}^3u_{22} - 72Q_{0,9}u_{01}u_{22}\\
&- 216Q_{0,9}u_{11}u_{12} - 72Q_{0,9}u_{12}u_{20}u_{30} - 36Q_{0,9}u_{20}^2u_{22} - 72Q_{1,11}u_{12}u_{30}\\
&- 72Q_{1,11}u_{20}u_{22} - Q_{1,13}u_{22} - 144Q_{1,8}u_{11}u_{22} - 144Q_{1,8}u_{12}u_{21}\\
&+ 195u_{01}^2u_{11}u_{22} + 288u_{01}^2u_{12}u_{21} - 156u_{01}u_{02}u_{10}u_{22}\\
&+ 10u_{01}u_{10}^2u_{22}u_{31} + 20u_{01}u_{10}u_{11}u_{22}u_{30} - 20u_{01}u_{10}u_{20}u_{21}u_{22}\\
&+ 432u_{01}u_{11}^2u_{12} + 432u_{01}u_{11}u_{12}u_{20}u_{30} - 82u_{01}u_{11}u_{20}^2u_{22}\\
&- 144u_{01}u_{12}u_{20}^2u_{21} + 144u_{01}u_{12}u_{20}^2u_{30}^2 + 96u_{01}u_{20}^3u_{22}u_{30}\\
&+ 12u_{02}u_{10}^2u_{22}u_{30} - 12u_{02}u_{10}u_{20}^2u_{22} - 2u_{10}^3u_{22}^2 + 6u_{10}^2u_{12}u_{20}u_{22}\\
&- 20u_{10}u_{11}^2u_{20}u_{22} - 216u_{11}^2u_{12}u_{20}^2 - 72u_{11}u_{12}u_{20}^3u_{30} - 42u_{11}u_{20}^4u_{22})\\
&+ \Omega(-288Q_{0,12}u_{10}u_{32} + 288Q_{0,12}u_{12}u_{30} + 432Q_{0,7}^2u_{22}\\
&- 144Q_{0,7}u_{01}u_{12}u_{40} - 144Q_{0,7}u_{01}u_{20}u_{32} + 576Q_{0,7}u_{01}u_{22}u_{30}\\
&+ 432Q_{0,7}u_{02}u_{12} - 288Q_{0,7}u_{10}u_{11}u_{32} - 288Q_{0,7}u_{10}u_{12}u_{31}\\
&- 576Q_{0,7}u_{10}u_{21}u_{22} - 144Q_{0,7}u_{11}u_{12}u_{30} - 864Q_{0,7}u_{11}u_{20}u_{22}\\
&- 72Q_{0,7}u_{12}u_{20}^2u_{40} - 720Q_{0,7}u_{12}u_{20}u_{21} - 144Q_{0,7}u_{12}u_{20}u_{30}^2\\
&- 24Q_{0,7}u_{20}^3u_{32} - 144Q_{0,7}u_{20}^2u_{22}u_{30} + 72Q_{0,9}u_{01}u_{32} + 432Q_{0,9}u_{11}u_{22}\\
&+ 72Q_{0,9}u_{12}u_{20}u_{40} + 360Q_{0,9}u_{12}u_{21} + 72Q_{0,9}u_{12}u_{30}^2 + 36Q_{0,9}u_{20}^2u_{32}\\
&+ 144Q_{0,9}u_{20}u_{22}u_{30} + 72Q_{1,11}u_{12}u_{40} + 72Q_{1,11}u_{20}u_{32}\\
&+ 144Q_{1,11}u_{22}u_{30} + Q_{1,13}u_{32} + 144Q_{1,8}u_{11}u_{32} + 144Q_{1,8}u_{12}u_{31}\\
&+ 288Q_{1,8}u_{21}u_{22} - 195u_{01}^2u_{11}u_{32} - 216u_{01}^2u_{12}u_{30}u_{40} - 288u_{01}^2u_{12}u_{31}\\
&- 576u_{01}^2u_{21}u_{22} + 156u_{01}u_{02}u_{10}u_{32} + 144u_{01}u_{02}u_{12}u_{30} - 10u_{01}u_{10}^2u_{31}u_{32}\\
&- 20u_{01}u_{10}u_{11}u_{30}u_{32} + 20u_{01}u_{10}u_{20}u_{21}u_{32} - 864u_{01}u_{11}^2u_{22}\\
&- 1440u_{01}u_{11}u_{12}u_{21} - 432u_{01}u_{11}u_{12}u_{30}^2 + 82u_{01}u_{11}u_{20}^2u_{32}\\
&- 864u_{01}u_{11}u_{20}u_{22}u_{30} - 144u_{01}u_{12}u_{20}^2u_{30}u_{40} + 144u_{01}u_{12}u_{20}^2u_{31}\\
&- 720u_{01}u_{12}u_{20}u_{21}u_{30} - 144u_{01}u_{12}u_{20}u_{30}^3 - 96u_{01}u_{20}^3u_{30}u_{32}\\
&+ 288u_{01}u_{20}^2u_{21}u_{22} - 288u_{01}u_{20}^2u_{22}u_{30}^2 - 12u_{02}u_{10}^2u_{30}u_{32}\\
&+ 12u_{02}u_{10}u_{20}^2u_{32} + 2u_{10}^3u_{22}u_{32} - 6u_{10}^2u_{12}u_{20}u_{32} + 20u_{10}u_{11}^2u_{20}u_{32}\\
&- 216u_{11}^3u_{12} + 144u_{11}^2u_{12}u_{20}u_{30} + 432u_{11}^2u_{20}^2u_{22}\\
&+ 72u_{11}u_{12}u_{20}^3u_{40} + 360u_{11}u_{12}u_{20}^2u_{21} + 72u_{11}u_{12}u_{20}^2u_{30}^2\\
&+ 42u_{11}u_{20}^4u_{32} + 144u_{11}u_{20}^3u_{22}u_{30})
\end{aligned}
$$

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\begin{aligned}
D_x\Omega(108) &= -\Omega_{yy}y + \Omega_y(u_{40}y + 1) + \Omega(-u_{40} + u_{41}y)\\
D_y\Omega(108) &= \Omega_{yy}u_{30}y - 2\Omega_{xy}u_{21}y + \Omega_{xx}u_{12}y + 2\Omega_y u_{31}y - \Omega_x u_{22}y + \Omega u_{32}y\\
D_x\Omega(109) &= -\Omega_{yy}x + \Omega_y(u_{30} + u_{40}x) + \Omega(3u_{31} + u_{41}x)\\
D_y\Omega(109) &= \Omega_{yy}u_{30}x - 2\Omega_{xy}u_{21}x + \Omega_{xx}u_{12}x + 2\Omega_y(u_{21} + u_{31}x)\\
&\quad - \Omega_x(u_{12} + u_{22}x) + \Omega(2u_{22} + u_{32}x)\\
D_x\Omega(110) &= \Omega_{yy}(3u_{10} - u_{20}x) + \Omega_y(-3u_{10}u_{40} - 3u_{11} - 2u_{20}u_{30} + u_{20}u_{40}x\\
&\quad + u_{21}x + u_{30}^2x) + \Omega(-3u_{10}u_{41} + 3u_{11}u_{40} + 3u_{12} - 6u_{20}u_{31} + u_{20}u_{41}x\\
&\quad + u_{21}u_{40}x - u_{22}x + 2u_{30}u_{31}x)\\
D_y\Omega(110) &= \Omega_{yy}u_{30}(-3u_{10} + u_{20}x)\\
&\quad + 2\Omega_{xy}u_{21}(3u_{10} - u_{20}x)
\end{aligned}
$$

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\begin{aligned}
&+ \Omega_{xx} u_{12}(-3u_{10} + u_{20}x) \\
&+ 2\Omega_y(-3u_{10}u_{31} - 2u_{20}u_{21} + u_{20}u_{31}x + u_{21}u_{30}x) \\
&+ \Omega_x(3u_{10}u_{22} + 2u_{12}u_{20} - u_{12}u_{30}x - u_{20}u_{22}x) \\
&+ \Omega(-3u_{10}u_{32} - u_{12}u_{30} + u_{12}u_{40}x - 4u_{20}u_{22} + u_{20}u_{32}x + 2u_{22}u_{30}x) \\
D_x\Omega(111) &= -\Omega_{yy}(u_{10} + u_{11}y) \\
&+ \Omega_y(u_{10}u_{40} + u_{11}u_{40}y + 2u_{11} + u_{12}y + u_{20}u_{30} + u_{21}u_{30}y) \\
&+ \Omega(u_{10}u_{41} - 2u_{11}u_{40} + u_{11}u_{41}y - 3u_{12} + 3u_{20}u_{31} + 3u_{21}u_{31}y) \\
D_y\Omega(111) &= \Omega_{yy}u_{30}(u_{10} + u_{11}y) \\
&- 2\Omega_{xy}u_{21}(u_{10} + u_{11}y) \\
&+ \Omega_{xx}u_{12}(u_{10} + u_{11}y) \\
&+ 2\Omega_y(u_{10}u_{31} + u_{11}u_{31}y + u_{20}u_{21} + u_{21}^2 y) \\
&- \Omega_x(u_{10}u_{22} + u_{11}u_{22}y + u_{12}u_{20} + u_{12}u_{21}y) \\
&+ \Omega(u_{10}u_{32} + u_{11}u_{32}y + u_{12}u_{30} + u_{12}u_{31}y + 2u_{20}u_{22} + 2u_{21}u_{22}y) \\
D_x\Omega(112) &= (\Omega_{yy}(-2u_{01} - 4u_{11}x - u_{20}^2) + \Omega_y(2u_{01}u_{40} + 2u_{02} + 6u_{11}u_{30} \\
&+ 4u_{11}u_{40}x + 4u_{12}x + u_{20}^2u_{40} + 2u_{20}u_{21} + 2u_{20}u_{30}^2 + 4u_{21}u_{30}x) \\
&+ \Omega(2u_{01}u_{41} - 2u_{02}u_{40} + 18u_{11}u_{31} + 4u_{11}u_{41}x - 4u_{12}u_{30} + u_{20}^2u_{41} \\
&+ 2u_{20}u_{21}u_{40} - 2u_{20}u_{22} + 4u_{20}u_{30}u_{31} + 16u_{21}^2 + 2u_{21}u_{30}^2 + 12u_{21}u_{31}x))/4 \\
D_y\Omega(112) &= (\Omega_{yy}u_{30}(2u_{01} + 4u_{11}x + u_{20}^2) \\
&+ 2\Omega_{xy}u_{21}(-2u_{01} - 4u_{11}x - u_{20}^2) \\
&+ \Omega_{xx}u_{12}(2u_{01} + 4u_{11}x + u_{20}^2) \\
&+ 2\Omega_y(2u_{01}u_{31} + 6u_{11}u_{21} + 4u_{11}u_{31}x + u_{20}^2u_{31} + 2u_{20}u_{21}u_{30} + 4u_{21}^2x) \\
&+ \Omega_x(-2u_{01}u_{22} - 6u_{11}u_{12} - 4u_{11}u_{22}x - 2u_{12}u_{20}u_{30} - 4u_{12}u_{21}x - u_{20}^2u_{22}) \\
&+ \Omega(2u_{01}u_{32} + 12u_{11}u_{22} + 4u_{11}u_{32}x + 2u_{12}u_{20}u_{40} + 10u_{12}u_{21} + 2u_{12}u_{30}^2 \\
&+ 4u_{12}u_{31}x + u_{20}^2u_{32} + 4u_{20}u_{22}u_{30} + 8u_{21}u_{22}x))/4 \\
D_x\Omega(113) &= \Omega_{yy}(7Q_{2,5} + 15Q_{1,3}u_{20} + 18Q_{1,6}x + 48uu_{11} - 10u_{01}u_{10} - 6u_{01}u_{20}x \\
&- 12u_{10}u_{11}x + 9u_{10}u_{20}^2 - u_{20}^3x) + \Omega_y(-7Q_{2,5}u_{40} - 30Q_{0,7}u_{20} \\
&- 18Q_{0,7}u_{30}x + 21Q_{0,9} - 15Q_{1,3}u_{20}u_{40} - 15Q_{1,3}u_{21} - 15Q_{1,3}u_{30}^2 \\
&+ 108Q_{1,6}u_{30} - 18Q_{1,6}u_{40}x - 48uu_{11}u_{40} - 48uu_{12} - 48uu_{21}u_{30} \\
&+ 10u_{01}u_{10}u_{40} - 24u_{01}u_{11} - 30u_{01}u_{20}u_{30} + 6u_{01}u_{20}u_{40}x + 6u_{01}u_{21}x \\
&- 12u_{01}u_{30}^2x - 18u_{02}u_{10} + 6u_{02}u_{20}x - 54u_{10}u_{11}u_{30} + 12u_{10}u_{11}u_{40}x \\
&+ 12u_{10}u_{12}x - 9u_{10}u_{20}^2u_{40} - 18u_{10}u_{20}u_{21} - 18u_{10}u_{20}u_{30}^2 + 12u_{10}u_{21}u_{30}x \\
&+ 3u_{11}^2x + 21u_{11}u_{20}^2 + 18u_{11}u_{20}u_{30}x + u_{20}^3u_{40}x + 3u_{20}^2u_{21}x + 3u_{20}^2u_{30}^2x) \\
&+ \Omega(-7Q_{2,5}u_{41} + 30Q_{0,7}u_{20}u_{40} + 240Q_{0,7}u_{21} + 30Q_{0,7}u_{30}^2 - 54Q_{0,7}u_{31}x \\
&- 21Q_{0,9}u_{40} - 15Q_{1,3}u_{20}u_{41} - 15Q_{1,3}u_{21}u_{40} + 15Q_{1,3}u_{22} - 30Q_{1,3}u_{30}u_{31} \\
&+ 324Q_{1,6}u_{31} - 18Q_{1,6}u_{41}x - 48uu_{11}u_{41} - 144uu_{21}u_{31} + 10u_{01}u_{10}u_{41} \\
&+ 24u_{01}u_{11}u_{40} + 72u_{01}u_{12} + 30u_{01}u_{20}u_{30}u_{40} + 6u_{01}u_{20}u_{41}x \\
&+ 240u_{01}u_{21}u_{30} + 6u_{01}u_{21}u_{40}x - 6u_{01}u_{22}x + 30u_{01}u_{30}^3 - 42u_{01}u_{30}u_{31}x \\
&+ 18u_{02}u_{10}u_{40} - 6u_{02}u_{20}u_{40}x - 48u_{02}u_{21}x - 6u_{02}u_{30}^2x - 162u_{10}u_{11}u_{31} \\
&+ 12u_{10}u_{11}u_{41}x + 36u_{10}u_{12}u_{30} - 9u_{10}u_{20}^2u_{41} - 18u_{10}u_{20}u_{21}u_{40} \\
&+ 18u_{10}u_{20}u_{22} - 36u_{10}u_{20}u_{30}u_{31} - 144u_{10}u_{21}^2 - 18u_{10}u_{21}u_{30}^2 \\
&+ 36u_{10}u_{21}u_{31}x - 3u_{11}^2u_{40}x - 18u_{11}u_{12}x - 21u_{11}u_{20}^2u_{40} - 96u_{11}u_{20}u_{21} \\
&- 12u_{11}u_{20}u_{30}^2 + 54u_{11}u_{20}u_{31}x - 12u_{12}u_{20}u_{30}x + u_{20}^3u_{41}x + 3u_{20}^2u_{21}u_{40}x
\end{aligned}
$$

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\begin{aligned}
&- 3u_{20}^2u_{22}x + 6u_{20}^2u_{30}u_{31}x + 48u_{20}u_{21}^2x + 6u_{20}u_{21}u_{30}^2x)\\
D_y\Omega(113) &= \Omega_{yy}u_{30}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 18Q_{1,6}x - 48uu_{11} + 10u_{01}u_{10}\\
&+ 6u_{01}u_{20}x + 12u_{10}u_{11}x - 9u_{10}u_{20}^2 + u_{20}^3x) + 2\Omega_{xy}u_{21}(7Q_{2,5} + 15Q_{1,3}u_{20}\\
&+ 18Q_{1,6}x + 48uu_{11} - 10u_{01}u_{10} - 6u_{01}u_{20}x - 12u_{10}u_{11}x + 9u_{10}u_{20}^2 - u_{20}^3x)\\
&+ \Omega_{xx}u_{12}(-7Q_{2,5} - 15Q_{1,3}u_{20} - 18Q_{1,6}x - 48uu_{11} + 10u_{01}u_{10}\\
&+ 6u_{01}u_{20}x + 12u_{10}u_{11}x - 9u_{10}u_{20}^2 + u_{20}^3x)\\
&+ 2\Omega_y(-7Q_{2,5}u_{31} - 18Q_{0,7}u_{21}x - 15Q_{1,3}u_{20}u_{31} - 15Q_{1,3}u_{21}u_{30}\\
&+ 108Q_{1,6}u_{21} - 18Q_{1,6}u_{31}x - 48uu_{11}u_{31} - 48uu_{21}^2 + 10u_{01}u_{10}u_{31}\\
&+ 6u_{01}u_{20}u_{31}x - 12u_{01}u_{21}u_{30}x - 54u_{10}u_{11}u_{21} + 12u_{10}u_{11}u_{31}x\\
&- 9u_{10}u_{20}^2u_{31} - 18u_{10}u_{20}u_{21}u_{30} + 12u_{10}u_{21}^2x + 18u_{11}u_{20}u_{21}x\\
&+ u_{20}^3u_{31}x + 3u_{20}^2u_{21}u_{30}x) + \Omega_x(7Q_{2,5}u_{22} + 18Q_{0,7}u_{12}x + 15Q_{1,3}u_{12}u_{30}\\
&+ 15Q_{1,3}u_{20}u_{22} - 108Q_{1,6}u_{12} + 18Q_{1,6}u_{22}x + 48uu_{11}u_{22} + 48uu_{12}u_{21}\\
&- 10u_{01}u_{10}u_{22} + 12u_{01}u_{12}u_{30}x - 6u_{01}u_{20}u_{22}x + 54u_{10}u_{11}u_{12}\\
&- 12u_{10}u_{11}u_{22}x + 18u_{10}u_{12}u_{20}u_{30} - 12u_{10}u_{12}u_{21}x + 9u_{10}u_{20}^2u_{22}\\
&- 18u_{11}u_{12}u_{20}x - 3u_{12}u_{20}^2u_{30}x - u_{20}^3u_{22}x) + \Omega(-7Q_{2,5}u_{32} + 90Q_{0,7}u_{12}\\
&- 36Q_{0,7}u_{22}x - 15Q_{1,3}u_{12}u_{40} - 15Q_{1,3}u_{20}u_{32} - 30Q_{1,3}u_{22}u_{30}\\
&+ 216Q_{1,6}u_{22} - 18Q_{1,6}u_{32}x - 48uu_{11}u_{32} - 48uu_{12}u_{31} - 96uu_{21}u_{22}\\
&+ 10u_{01}u_{10}u_{32} + 66u_{01}u_{12}u_{30} + 6u_{01}u_{12}u_{40}x + 6u_{01}u_{20}u_{32}x\\
&- 24u_{01}u_{22}u_{30}x - 18u_{02}u_{12}x - 108u_{10}u_{11}u_{22} + 12u_{10}u_{11}u_{32}x\\
&- 18u_{10}u_{12}u_{20}u_{40} - 90u_{10}u_{12}u_{21} - 18u_{10}u_{12}u_{30}^2 + 12u_{10}u_{12}u_{31}x\\
&- 9u_{10}u_{20}^2u_{32} - 36u_{10}u_{20}u_{22}u_{30} + 24u_{10}u_{21}u_{22}x - 36u_{11}u_{12}u_{20}\\
&+ 6u_{11}u_{12}u_{30}x + 36u_{11}u_{20}u_{22}x + 3u_{12}u_{20}^2u_{40}x + 30u_{12}u_{20}u_{21}x\\
&+ 6u_{12}u_{20}u_{30}^2x + u_{20}^3u_{32}x + 6u_{20}^2u_{22}u_{30}x)\\
D_x\Omega(114) &= \Omega_{yy}(-Q_{2,5} - 2Q_{0,7}u_{20}y + Q_{0,9}y - 2Q_{1,3}u_{20} - 4uu_{11} + 2u_{01}u_{10}\\
&- 4u_{01}u_{11}y - 2u_{01}u_{20}u_{30}y - u_{10}u_{20}^2 + u_{11}u_{20}^2y) + \Omega_y(Q_{2,5}u_{40}\\
&+ 2Q_{0,7}u_{20}u_{40}y + 6Q_{0,7}u_{20} + 2Q_{0,7}u_{21}y + 2Q_{0,7}u_{30}^2y - Q_{0,9}u_{40}y - 4Q_{0,9}\\
&+ 2Q_{1,3}u_{20}u_{40} + 2Q_{1,3}u_{21} + 2Q_{1,3}u_{30}^2 - 18Q_{1,6}u_{30} + 4uu_{11}u_{40} + 4uu_{12}\\
&+ 4uu_{21}u_{30} - 2u_{01}u_{10}u_{40} + 4u_{01}u_{11}u_{40}y + 4u_{01}u_{11} + 4u_{01}u_{12}y\\
&+ 2u_{01}u_{20}u_{30}u_{40}y + 6u_{01}u_{20}u_{30} + 6u_{01}u_{21}u_{30}y + 2u_{01}u_{30}^3y + 2u_{02}u_{10}\\
&+ 2u_{02}u_{11}y + 6u_{10}u_{11}u_{30} + u_{10}u_{20}^2u_{40} + 2u_{10}u_{20}u_{21} + 2u_{10}u_{20}u_{30}^2\\
&+ 3u_{11}^2u_{30}y - u_{11}u_{20}^2u_{40}y - 4u_{11}u_{20}^2) + \Omega(Q_{2,5}u_{41} - 6Q_{0,7}u_{20}u_{40}\\
&+ 2Q_{0,7}u_{20}u_{41}y + 2Q_{0,7}u_{21}u_{40}y - 48Q_{0,7}u_{21} - 2Q_{0,7}u_{22}y\\
&- 6Q_{0,7}u_{30}^2 + 4Q_{0,7}u_{30}u_{31}y + 4Q_{0,9}u_{40} - Q_{0,9}u_{41}y + 2Q_{1,3}u_{20}u_{41}\\
&+ 2Q_{1,3}u_{21}u_{40} - 2Q_{1,3}u_{22} + 4Q_{1,3}u_{30}u_{31} - 54Q_{1,6}u_{31} + 4uu_{11}u_{41}\\
&+ 12uu_{21}u_{31} - 2u_{01}u_{10}u_{41} - 4u_{01}u_{11}u_{40} + 4u_{01}u_{11}u_{41}y - 12u_{01}u_{12}\\
&- 6u_{01}u_{20}u_{30}u_{40} + 2u_{01}u_{20}u_{30}u_{41}y + 2u_{01}u_{21}u_{30}u_{40}y\\
&- 48u_{01}u_{21}u_{30} + 12u_{01}u_{21}u_{31}y - 2u_{01}u_{22}u_{30}y - 6u_{01}u_{30}^3 + 4u_{01}u_{30}^2u_{31}y\\
&- 2u_{02}u_{10}u_{40} - 2u_{02}u_{11}u_{40}y - 6u_{02}u_{12}y + 18u_{10}u_{11}u_{31} - 4u_{10}u_{12}u_{30}\\
&+ u_{10}u_{20}^2u_{41} + 2u_{10}u_{20}u_{21}u_{40} - 2u_{10}u_{20}u_{22} + 4u_{10}u_{20}u_{30}u_{31} + 16u_{10}u_{21}^2\\
&+ 2u_{10}u_{21}u_{30}^2 + 9u_{11}^2u_{31}y - 4u_{11}u_{12}u_{30}y + 4u_{11}u_{20}^2u_{40} - u_{11}u_{20}^2u_{41}y\\
&+ 16u_{11}u_{20}u_{21} + 2u_{11}u_{20}u_{30}^2 + 16u_{11}u_{21}^2y + 2u_{11}u_{21}u_{30}^2y)\\
D_y\Omega(114) &= \Omega_{yy}u_{30}(Q_{2,5} + 2Q_{0,7}u_{20}y - Q_{0,9}y + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10}
\end{aligned}
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\begin{aligned}
&+ 4u_{01}u_{11}y + 2u_{01}u_{20}u_{30}y + u_{10}u_{20}^2 - u_{11}u_{20}^2y)\\
&+ 2\Omega_{xy}u_{21}(-Q_{2,5} - 2Q_{0,7}u_{20}y + Q_{0,9}y - 2Q_{1,3}u_{20} - 4uu_{11} + 2u_{01}u_{10}\\
&- 4u_{01}u_{11}y - 2u_{01}u_{20}u_{30}y - u_{10}u_{20}^2 + u_{11}u_{20}^2y)\\
&+ \Omega_{xx}u_{12}(Q_{2,5} + 2Q_{0,7}u_{20}y - Q_{0,9}y + 2Q_{1,3}u_{20} + 4uu_{11} - 2u_{01}u_{10}\\
&+ 4u_{01}u_{11}y + 2u_{01}u_{20}u_{30}y + u_{10}u_{20}^2 - u_{11}u_{20}^2y)\\
&+ 2\Omega_y(Q_{2,5}u_{31} + 2Q_{0,7}u_{20}u_{31}y + 2Q_{0,7}u_{21}u_{30}y - Q_{0,9}u_{31}y + 2Q_{1,3}u_{20}u_{31}\\
&+ 2Q_{1,3}u_{21}u_{30} - 18Q_{1,6}u_{21} + 4uu_{11}u_{31} + 4uu_{21}^2 - 2u_{01}u_{10}u_{31}\\
&+ 4u_{01}u_{11}u_{31}y + 2u_{01}u_{20}u_{30}u_{31}y + 4u_{01}u_{21}^2y + 2u_{01}u_{21}u_{30}^2y + 6u_{10}u_{11}u_{21}\\
&+ u_{10}u_{20}^2u_{31} + 2u_{10}u_{20}u_{21}u_{30} + 3u_{11}^2u_{21}y - u_{11}u_{20}^2u_{31}y)\\
&+ \Omega_x(-Q_{2,5}u_{22} - 2Q_{0,7}u_{12}u_{30}y - 2Q_{0,7}u_{20}u_{22}y + Q_{0,9}u_{22}y\\
&- 2Q_{1,3}u_{12}u_{30} - 2Q_{1,3}u_{20}u_{22} + 18Q_{1,6}u_{12} - 4uu_{11}u_{22} - 4uu_{12}u_{21}\\
&+ 2u_{01}u_{10}u_{22} - 4u_{01}u_{11}u_{22}y - 4u_{01}u_{12}u_{21}y - 2u_{01}u_{12}u_{30}^2y - 2u_{01}u_{20}u_{22}u_{30}y\\
&- 6u_{10}u_{11}u_{12} - 2u_{10}u_{12}u_{20}u_{30} - u_{10}u_{20}^2u_{22} - 3u_{11}^2u_{12}y + u_{11}u_{20}^2u_{22}y)\\
&+ \Omega(Q_{2,5}u_{32} + 2Q_{0,7}u_{12}u_{40}y - 18Q_{0,7}u_{12} + 2Q_{0,7}u_{20}u_{32}y + 4Q_{0,7}u_{22}u_{30}y\\
&- Q_{0,9}u_{32}y + 2Q_{1,3}u_{12}u_{40} + 2Q_{1,3}u_{20}u_{32} + 4Q_{1,3}u_{22}u_{30} - 36Q_{1,6}u_{22}\\
&+ 4uu_{11}u_{32} + 4uu_{12}u_{31} + 8uu_{21}u_{22} - 2u_{01}u_{10}u_{32} + 4u_{01}u_{11}u_{32}y\\
&+ 2u_{01}u_{12}u_{30}u_{40}y - 14u_{01}u_{12}u_{30} + 4u_{01}u_{12}u_{31}y\\
&+ 2u_{01}u_{20}u_{30}u_{32}y + 8u_{01}u_{21}u_{22}y + 4u_{01}u_{22}u_{30}^2y + 2u_{02}u_{12}u_{30}y\\
&+ 12u_{10}u_{11}u_{22} + 2u_{10}u_{12}u_{20}u_{40} + 10u_{10}u_{12}u_{21} + 2u_{10}u_{12}u_{30}^2\\
&+ u_{10}u_{20}^2u_{32} + 4u_{10}u_{20}u_{22}u_{30}\\
&+ 6u_{11}^2u_{22}y + 6u_{11}u_{12}u_{20} + 10u_{11}u_{12}u_{21}y + 2u_{11}u_{12}u_{30}^2y - u_{11}u_{20}^2u_{32}y)\\
D_x\Omega(115) &= (\Omega_{yy}(-24Q_{0,7}u_{10} - 48Q_{0,7}u_{20}x + 24Q_{0,9}x + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20}\\
&+ 12Q_{1,8} - 12u_{01}^2 - 96u_{01}u_{11}x - 48u_{01}u_{20}u_{30}x + 24u_{11}u_{20}^2x - u_{20}^4)\\
&+ \Omega_y(-48Q_{0,12} + 24Q_{0,7}u_{10}u_{40} - 24Q_{0,7}u_{11} + 48Q_{0,7}u_{20}u_{40}x\\
&+ 48Q_{0,7}u_{21}x + 48Q_{0,7}u_{30}^2x - 36Q_{0,9}u_{30} - 24Q_{0,9}u_{40}x - 24Q_{1,3}u_{11}u_{40}\\
&- 24Q_{1,3}u_{12} - 24Q_{1,3}u_{21}u_{30} - 72Q_{1,6}u_{20}u_{40} - 72Q_{1,6}u_{21} - 72Q_{1,6}u_{30}^2\\
&- 12Q_{1,8}u_{40} + 12u_{01}^2u_{40} + 24u_{01}u_{02} + 48u_{01}u_{11}u_{30} + 96u_{01}u_{11}u_{40}x\\
&+ 96u_{01}u_{12}x + 24u_{01}u_{20}u_{21} + 24u_{01}u_{20}u_{30}^2 + 48u_{01}u_{20}u_{30}u_{40}x\\
&+ 144u_{01}u_{21}u_{30}x + 48u_{01}u_{30}^3x + 48u_{02}u_{11}x + 12u_{02}u_{20}^2 - 12u_{11}^2u_{20}\\
&+ 72u_{11}^2u_{30}x - 24u_{11}u_{20}^2u_{40}x + u_{20}^4u_{40} + 4u_{20}^3u_{21} + 4u_{20}^3u_{30}^2)\\
&+ \Omega(48Q_{0,12}u_{40} + 24Q_{0,7}u_{10}u_{41} + 24Q_{0,7}u_{11}u_{40} + 72Q_{0,7}u_{12}\\
&+ 48Q_{0,7}u_{20}u_{41}x + 48Q_{0,7}u_{21}u_{40}x - 48Q_{0,7}u_{22}x + 96Q_{0,7}u_{30}u_{31}x\\
&- 108Q_{0,9}u_{31} - 24Q_{0,9}u_{41}x - 24Q_{1,3}u_{11}u_{41} - 72Q_{1,3}u_{21}u_{31}\\
&- 72Q_{1,6}u_{20}u_{41} - 72Q_{1,6}u_{21}u_{40} + 72Q_{1,6}u_{22} - 144Q_{1,6}u_{30}u_{31}\\
&- 12Q_{1,8}u_{41} + 12u_{01}^2u_{41} - 24u_{01}u_{02}u_{40} + 24u_{01}u_{11}u_{30}u_{40}\\
&+ 216u_{01}u_{11}u_{31} + 96u_{01}u_{11}u_{41}x + 24u_{01}u_{12}u_{30} + 24u_{01}u_{20}u_{21}u_{40}\\
&- 24u_{01}u_{20}u_{22} + 48u_{01}u_{20}u_{30}u_{31} + 48u_{01}u_{20}u_{30}u_{41}x\\
&+ 192u_{01}u_{21}^2 + 24u_{01}u_{21}u_{30}^2 + 48u_{01}u_{21}u_{30}u_{40}x + 288u_{01}u_{21}u_{31}x\\
&- 48u_{01}u_{22}u_{30}x + 96u_{01}u_{30}^2u_{31}x\\
&- 48u_{02}u_{11}u_{40}x - 144u_{02}u_{12}x - 12u_{02}u_{20}^2u_{40} - 192u_{02}u_{20}u_{21}\\
&- 24u_{02}u_{20}u_{30}^2 + 12u_{11}^2u_{20}u_{40}\\
&+ 288u_{11}^2u_{21} + 36u_{11}^2u_{30}^2 + 216u_{11}^2u_{31}x - 72u_{11}u_{12}u_{20} - 96u_{11}u_{12}u_{30}x
\end{aligned}
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\begin{aligned}
&- 24u_{11}u_{20}^2u_{41}x + 384u_{11}u_{21}^2x + 48u_{11}u_{21}u_{30}^2x - 24u_{12}u_{20}^2u_{30}\\
&+ u_{20}^4u_{41} + 4u_{20}^3u_{21}u_{40} - 4u_{20}^3u_{22}\\
&+ 8u_{20}^3u_{30}u_{31} + 96u_{20}^2u_{21}^2 + 12u_{20}^2u_{21}u_{30}^2))/24\\
D_y\Omega(115) = {}&(\Omega_{yy}u_{30}(24Q_{0,7}u_{10} + 48Q_{0,7}u_{20}x - 24Q_{0,9}x - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20}\\
&- 12Q_{1,8} + 12u_{01}^2 + 96u_{01}u_{11}x + 48u_{01}u_{20}u_{30}x - 24u_{11}u_{20}^2x + u_{20}^4)\\
&+ 2\Omega_{xy}u_{21}(-24Q_{0,7}u_{10} - 48Q_{0,7}u_{20}x + 24Q_{0,9}x + 24Q_{1,3}u_{11} + 72Q_{1,6}u_{20} + 12Q_{1,8}\\
&- 12u_{01}^2 - 96u_{01}u_{11}x - 48u_{01}u_{20}u_{30}x + 24u_{11}u_{20}^2x - u_{20}^4)\\
&+ \Omega_{xx}u_{12}(24Q_{0,7}u_{10} + 48Q_{0,7}u_{20}x - 24Q_{0,9}x - 24Q_{1,3}u_{11} - 72Q_{1,6}u_{20}\\
&- 12Q_{1,8} + 12u_{01}^2 + 96u_{01}u_{11}x + 48u_{01}u_{20}u_{30}x - 24u_{11}u_{20}^2x + u_{20}^4)\\
&+ 2\Omega_y(24Q_{0,7}u_{10}u_{31} + 48Q_{0,7}u_{20}u_{31}x + 48Q_{0,7}u_{21}u_{30}x - 36Q_{0,9}u_{21}\\
&- 24Q_{0,9}u_{31}x - 24Q_{1,3}u_{11}u_{31} - 24Q_{1,3}u_{21}^2 - 72Q_{1,6}u_{20}u_{31}\\
&- 72Q_{1,6}u_{21}u_{30} - 12Q_{1,8}u_{31} + 12u_{01}^2u_{31} + 72u_{01}u_{11}u_{21} + 96u_{01}u_{11}u_{31}x\\
&+ 24u_{01}u_{20}u_{21}u_{30} + 48u_{01}u_{20}u_{30}u_{31}x + 96u_{01}u_{21}^2x\\
&+ 48u_{01}u_{21}u_{30}^2x + 72u_{11}^2u_{21}x - 24u_{11}u_{20}^2u_{31}x + u_{20}^4u_{31} + 4u_{20}^3u_{21}u_{30})\\
&+ \Omega_x(-24Q_{0,7}u_{10}u_{22} - 48Q_{0,7}u_{12}u_{30}x - 48Q_{0,7}u_{20}u_{22}x + 36Q_{0,9}u_{12}\\
&+ 24Q_{0,9}u_{22}x + 24Q_{1,3}u_{11}u_{22} + 24Q_{1,3}u_{12}u_{21} + 72Q_{1,6}u_{12}u_{30}\\
&+ 72Q_{1,6}u_{20}u_{22} + 12Q_{1,8}u_{22} - 12u_{01}^2u_{22} - 72u_{01}u_{11}u_{12} - 96u_{01}u_{11}u_{22}x\\
&- 24u_{01}u_{12}u_{20}u_{30} - 96u_{01}u_{12}u_{21}x - 48u_{01}u_{12}u_{30}^2x\\
&- 48u_{01}u_{20}u_{22}u_{30}x - 72u_{11}^2u_{12}x + 24u_{11}u_{20}^2u_{22}x - 4u_{12}u_{20}^3u_{30} - u_{20}^4u_{22})\\
&+ \Omega(24Q_{0,7}u_{10}u_{32} - 24Q_{0,7}u_{12}u_{30} + 48Q_{0,7}u_{12}u_{40}x + 48Q_{0,7}u_{20}u_{32}x\\
&+ 96Q_{0,7}u_{22}u_{30}x - 72Q_{0,9}u_{22} - 24Q_{0,9}u_{32}x - 24Q_{1,3}u_{11}u_{32} - 24Q_{1,3}u_{12}u_{31}\\
&- 48Q_{1,3}u_{21}u_{22} - 72Q_{1,6}u_{12}u_{40} - 72Q_{1,6}u_{20}u_{32} - 144Q_{1,6}u_{22}u_{30}\\
&- 12Q_{1,8}u_{32} + 12u_{01}^2u_{32} + 144u_{01}u_{11}u_{22} + 96u_{01}u_{11}u_{32}x\\
&+ 24u_{01}u_{12}u_{20}u_{40} + 120u_{01}u_{12}u_{21} + 48u_{01}u_{12}u_{30}u_{40}x + 96u_{01}u_{12}u_{31}x\\
&+ 48u_{01}u_{20}u_{22}u_{30} + 48u_{01}u_{20}u_{30}u_{32}x + 192u_{01}u_{21}u_{22}x\\
&+ 96u_{01}u_{22}u_{30}^2x - 72u_{02}u_{12}u_{20} + 48u_{02}u_{12}u_{30}x + 108u_{11}^2u_{12}\\
&+ 144u_{11}^2u_{22}x + 24u_{11}u_{12}u_{20}u_{30} + 240u_{11}u_{12}u_{21}x + 48u_{11}u_{12}u_{30}^2x\\
&- 24u_{11}u_{20}^2u_{32}x + 4u_{12}u_{20}^3u_{40} + 60u_{12}u_{20}^2u_{21} + 12u_{12}u_{20}^2u_{30}^2\\
&+ u_{20}^4u_{32} + 8u_{20}^3u_{22}u_{30}))/24\\
D_x\Omega(116) = {}&(\Omega_{yy}(-2u_{20}y + 3x^2)\\
&+ \Omega_y(2u_{20}u_{40}y + 2u_{20} + 2u_{21}y + 2u_{30}^2y - 6u_{30}x - 3u_{40}x^2)\\
&+ \Omega(-2u_{20}u_{40} + 2u_{20}u_{41}y + 2u_{21}u_{40}y - 16u_{21}\\
&- 2u_{22}y - 2u_{30}^2 + 4u_{30}u_{31}y - 18u_{31}x - 3u_{41}x^2))/2\\
D_y\Omega(116) = {}&(\Omega_{yy}u_{30}(2u_{20}y - 3x^2)\\
&+ 2\Omega_{xy}u_{21}(-2u_{20}y + 3x^2)\\
&+ \Omega_{xx}u_{12}(2u_{20}y - 3x^2)\\
&+ 2\Omega_y(2u_{20}u_{31}y + 2u_{21}u_{30}y - 6u_{21}x - 3u_{31}x^2)\\
&+ \Omega_x(-2u_{12}u_{30}y + 6u_{12}x - 2u_{20}u_{22}y + 3u_{22}x^2)\\
&+ \Omega(2u_{12}u_{40}y - 6u_{12} + 2u_{20}u_{32}y + 4u_{22}u_{30}y - 12u_{22}x - 3u_{32}x^2))/2\\
D_x\Omega(117) = {}&(\Omega_{yy}(-4u + 2u_{01}y - 2u_{10}x + 4u_{11}xy + u_{20}^2y + u_{20}x^2)\\
&+ \Omega_y(4uu_{40} - 2u_{01}u_{40}y + 2u_{01} - 2u_{02}y + 6u_{10}u_{30} + 2u_{10}u_{40}x\\
&- 6u_{11}u_{30}y - 4u_{11}u_{40}xy
\end{aligned}
$$

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\begin{aligned}
&- 2u_{11}x - 4u_{12}xy - u_{20}^2u_{40}y - u_{20}^2 - 2u_{20}u_{21}y - 2u_{20}u_{30}^2y - u_{20}u_{40}x^2\\
&- 4u_{21}u_{30}xy - u_{21}x^2 - u_{30}^2x^2)\\
&+ \Omega(4uu_{41} - 2u_{01}u_{40} - 2u_{01}u_{41}y + 2u_{02}u_{40}y + 18u_{10}u_{31} + 2u_{10}u_{41}x\\
&- 18u_{11}u_{31}y + 2u_{11}u_{40}x - 4u_{11}u_{41}xy + 4u_{12}u_{30}y + 6u_{12}x + u_{20}^2u_{40}\\
&- u_{20}^2u_{41}y - 2u_{20}u_{21}u_{40}y + 16u_{20}u_{21} + 2u_{20}u_{22}y + 2u_{20}u_{30}^2\\
&- 4u_{20}u_{30}u_{31}y - u_{20}u_{41}x^2 - 16u_{21}^2y - 2u_{21}u_{30}^2y - 12u_{21}u_{31}xy\\
&- u_{21}u_{40}x^2 + u_{22}x^2 - 2u_{30}u_{31}x^2))/2\\
D_y\Omega(117) = {}&(\Omega_{yy}u_{30}(4u - 2u_{01}y + 2u_{10}x - 4u_{11}xy - u_{20}^2y - u_{20}x^2)\\
&+ 2\Omega_{xy}u_{21}(-4u + 2u_{01}y - 2u_{10}x + 4u_{11}xy + u_{20}^2y + u_{20}x^2)\\
&+ \Omega_{xx}u_{12}(4u - 2u_{01}y + 2u_{10}x - 4u_{11}xy - u_{20}^2y - u_{20}x^2)\\
&+ 2\Omega_y(4uu_{31} - 2u_{01}u_{31}y + 6u_{10}u_{21} + 2u_{10}u_{31}x - 6u_{11}u_{21}y\\
&- 4u_{11}u_{31}xy - u_{20}^2u_{31}y - 2u_{20}u_{21}u_{30}y - u_{20}u_{31}x^2 - 4u_{21}^2xy - u_{21}u_{30}x^2)\\
&+ \Omega_x(-4uu_{22} + 2u_{01}u_{22}y - 6u_{10}u_{12} - 2u_{10}u_{22}x + 6u_{11}u_{12}y + 4u_{11}u_{22}xy\\
&+ 2u_{12}u_{20}u_{30}y + 4u_{12}u_{21}xy + u_{12}u_{30}x^2 + u_{20}^2u_{22}y + u_{20}u_{22}x^2)\\
&+ \Omega(4uu_{32} - 2u_{01}u_{32}y + 12u_{10}u_{22} + 2u_{10}u_{32}x - 12u_{11}u_{22}y\\
&- 4u_{11}u_{32}xy - 2u_{12}u_{20}u_{40}y + 6u_{12}u_{20} - 10u_{12}u_{21}y - 2u_{12}u_{30}^2y\\
&- 2u_{12}u_{30}x - 4u_{12}u_{31}xy - u_{12}u_{40}x^2 - u_{20}^2u_{32}y - 4u_{20}u_{22}u_{30}y\\
&- u_{20}u_{32}x^2 - 8u_{21}u_{22}xy - 2u_{22}u_{30}x^2))/2\\
D_x\Omega(118) = {}&(\Omega_{yy}(-4Q_{2,5}x - 24Q_{3,4} + 24Q_{0,7}u_{10}y + 48Q_{0,7}u_{20}xy - 24Q_{0,9}xy\\
&+ 36Q_{1,3}u_{10} - 24Q_{1,3}u_{11}y - 12Q_{1,3}u_{20}x - 72Q_{1,6}u_{20}y - 36Q_{1,6}x^2\\
&- 12Q_{1,8}y - 48uu_{01} - 96uu_{11}x - 24uu_{20}^2 + 12u_{01}^2y - 8u_{01}u_{10}x\\
&+ 96u_{01}u_{11}xy + 48u_{01}u_{20}u_{30}xy + 12u_{01}u_{20}x^2 + 36u_{10}^2u_{20} + 24u_{10}u_{11}x^2\\
&- 12u_{10}u_{20}^2x - 24u_{11}u_{20}^2xy + u_{20}^4y + 2u_{20}^3x^2) + \Omega_y(12Q_{2,5}u_{30} + 4Q_{2,5}u_{40}x\\
&+ 24Q_{3,4}u_{40} + 48Q_{0,12}y - 24Q_{0,7}u_{10}u_{40}y\\
&- 48Q_{0,7}u_{10} + 24Q_{0,7}u_{11}y - 48Q_{0,7}u_{20}u_{40}xy - 24Q_{0,7}u_{20}x - 48Q_{0,7}u_{21}xy\\
&- 48Q_{0,7}u_{30}^2xy + 36Q_{0,7}u_{30}x^2 + 36Q_{0,9}u_{30}y + 24Q_{0,9}u_{40}xy + 12Q_{0,9}x\\
&- 36Q_{1,3}u_{10}u_{40} + 24Q_{1,3}u_{11}u_{40}y + 12Q_{1,3}u_{11} + 24Q_{1,3}u_{12}y\\
&+ 12Q_{1,3}u_{20}u_{40}x + 24Q_{1,3}u_{21}u_{30}y + 12Q_{1,3}u_{21}x + 12Q_{1,3}u_{30}^2x\\
&+ 72Q_{1,6}u_{20}u_{40}y + 72Q_{1,6}u_{20} + 72Q_{1,6}u_{21}y + 72Q_{1,6}u_{30}^2y + 36Q_{1,6}u_{40}x^2\\
&+ 12Q_{1,8}u_{40}y - 12Q_{1,8} + 48uu_{01}u_{40} + 48uu_{02} + 144uu_{11}u_{30} + 96uu_{11}u_{40}x\\
&+ 96uu_{12}x + 24uu_{20}^2u_{40} + 48uu_{20}u_{21} + 48uu_{20}u_{30}^2 + 96uu_{21}u_{30}x\\
&- 12u_{01}^2u_{40}y - 12u_{01}^2 - 24u_{01}u_{02}y - 120u_{01}u_{10}u_{30} + 8u_{01}u_{10}u_{40}x\\
&- 48u_{01}u_{11}u_{30}y - 96u_{01}u_{11}u_{40}xy - 96u_{01}u_{12}xy - 24u_{01}u_{20}u_{21}y\\
&- 24u_{01}u_{20}u_{30}^2y - 48u_{01}u_{20}u_{30}u_{40}xy - 24u_{01}u_{20}u_{30}x\\
&- 12u_{01}u_{20}u_{40}x^2 - 144u_{01}u_{21}u_{30}xy - 12u_{01}u_{21}x^2 - 48u_{01}u_{30}^3xy\\
&+ 24u_{01}u_{30}^2x^2 + 24u_{02}u_{10}x - 48u_{02}u_{11}xy - 12u_{02}u_{20}^2y - 12u_{02}u_{20}x^2\\
&- 36u_{10}^2u_{20}u_{40} - 36u_{10}^2u_{21} - 36u_{10}^2u_{30}^2 + 72u_{10}u_{11}u_{30}x\\
&- 24u_{10}u_{11}u_{40}x^2 - 24u_{10}u_{12}x^2 + 12u_{10}u_{20}^2u_{40}x + 24u_{10}u_{20}u_{21}x\\
&+ 24u_{10}u_{20}u_{30}^2x - 24u_{10}u_{21}u_{30}x^2 + 12u_{11}^2u_{20}y - 72u_{11}^2u_{30}xy\\
&- 6u_{11}^2x^2 + 24u_{11}u_{20}^2u_{40}xy + 12u_{11}u_{20}^2x - 36u_{11}u_{20}u_{30}x^2\\
&- u_{20}^4u_{40}y - u_{20}^4 - 4u_{20}^3u_{21}y - 4u_{20}^3u_{30}^2y - 2u_{20}^3u_{40}x^2
\end{aligned}
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\begin{aligned}
&- 6u_{20}^2u_{21}x^2 - 6u_{20}^2u_{30}^2x^2)\\
&+ \Omega(36Q_{2,5}u_{31} + 4Q_{2,5}u_{41}x + 24Q_{3,4}u_{41} - 48Q_{0,12}u_{40}y + 48Q_{0,7}u_{10}u_{40}\\
&- 24Q_{0,7}u_{10}u_{41}y - 24Q_{0,7}u_{11}u_{40}y - 72Q_{0,7}u_{12}y + 24Q_{0,7}u_{20}u_{40}x\\
&- 48Q_{0,7}u_{20}u_{41}xy - 48Q_{0,7}u_{21}u_{40}xy + 192Q_{0,7}u_{21}x + 48Q_{0,7}u_{22}xy\\
&+ 24Q_{0,7}u_{30}^2x - 96Q_{0,7}u_{30}u_{31}xy + 108Q_{0,7}u_{31}x^2 + 108Q_{0,9}u_{31}y\\
&- 12Q_{0,9}u_{40}x + 24Q_{0,9}u_{41}xy - 36Q_{1,3}u_{10}u_{41} - 12Q_{1,3}u_{11}u_{40}\\
&+ 24Q_{1,3}u_{11}u_{41}y - 36Q_{1,3}u_{12} + 12Q_{1,3}u_{20}u_{41}x + 72Q_{1,3}u_{21}u_{31}y\\
&+ 12Q_{1,3}u_{21}u_{40}x - 12Q_{1,3}u_{22}x + 24Q_{1,3}u_{30}u_{31}x - 72Q_{1,6}u_{20}u_{40}\\
&+ 72Q_{1,6}u_{20}u_{41}y + 72Q_{1,6}u_{21}u_{40}y - 576Q_{1,6}u_{21} - 72Q_{1,6}u_{22}y\\
&- 72Q_{1,6}u_{30}^2 + 144Q_{1,6}u_{30}u_{31}y + 36Q_{1,6}u_{41}x^2 + 12Q_{1,8}u_{40}\\
&+ 12Q_{1,8}u_{41}y + 48uu_{01}u_{41} - 48uu_{02}u_{40} + 432uu_{11}u_{31} + 96uu_{11}u_{41}x\\
&- 96uu_{12}u_{30} + 24uu_{20}^2u_{41} + 48uu_{20}u_{21}u_{40} - 48uu_{20}u_{22} + 96uu_{20}u_{30}u_{31}\\
&+ 384uu_{21}^2 + 48uu_{21}u_{30}^2 + 288uu_{21}u_{31}x + 12u_{01}^2u_{40} - 12u_{01}^2u_{41}y\\
&+ 24u_{01}u_{02}u_{40}y + 72u_{01}u_{10}u_{30}u_{40} - 144u_{01}u_{10}u_{31} + 8u_{01}u_{10}u_{41}x\\
&- 24u_{01}u_{11}u_{30}u_{40}y - 216u_{01}u_{11}u_{31}y - 96u_{01}u_{11}u_{41}xy\\
&- 24u_{01}u_{12}u_{30}y - 24u_{01}u_{20}u_{21}u_{40}y - 192u_{01}u_{20}u_{21} + 24u_{01}u_{20}u_{22}y\\
&- 24u_{01}u_{20}u_{30}^2 - 48u_{01}u_{20}u_{30}u_{31}y + 24u_{01}u_{20}u_{30}u_{40}x\\
&- 48u_{01}u_{20}u_{30}u_{41}xy - 12u_{01}u_{20}u_{41}x^2 - 192u_{01}u_{21}^2y - 24u_{01}u_{21}u_{30}^2y\\
&- 48u_{01}u_{21}u_{30}u_{40}xy + 192u_{01}u_{21}u_{30}x - 288u_{01}u_{21}u_{31}xy - 12u_{01}u_{21}u_{40}x^2\\
&+ 48u_{01}u_{22}u_{30}xy + 12u_{01}u_{22}x^2 + 24u_{01}u_{30}^3x - 96u_{01}u_{30}^2u_{31}xy\\
&+ 84u_{01}u_{30}u_{31}x^2 - 24u_{02}u_{10}u_{40}x + 48u_{02}u_{11}u_{40}xy\\
&+ 144u_{02}u_{12}xy + 12u_{02}u_{20}^2u_{40}y + 192u_{02}u_{20}u_{21}y + 24u_{02}u_{20}u_{30}^2y\\
&+ 12u_{02}u_{20}u_{40}x^2 + 96u_{02}u_{21}x^2 + 12u_{02}u_{30}^2x^2 - 36u_{10}^2u_{20}u_{41}\\
&- 36u_{10}^2u_{21}u_{40} + 36u_{10}^2u_{22} - 72u_{10}^2u_{30}u_{31} + 576u_{10}u_{11}u_{21}\\
&+ 72u_{10}u_{11}u_{30}^2 + 216u_{10}u_{11}u_{31}x - 24u_{10}u_{11}u_{41}x^2 - 48u_{10}u_{12}u_{30}x\\
&+ 12u_{10}u_{20}^2u_{41}x + 24u_{10}u_{20}u_{21}u_{40}x - 24u_{10}u_{20}u_{22}x + 48u_{10}u_{20}u_{30}u_{31}x\\
&+ 192u_{10}u_{21}^2x + 24u_{10}u_{21}u_{30}^2x - 72u_{10}u_{21}u_{31}x^2 - 12u_{11}^2u_{20}u_{40}y\\
&- 288u_{11}^2u_{21}y - 36u_{11}^2u_{30}^2y - 216u_{11}^2u_{31}xy + 6u_{11}^2u_{40}x^2\\
&+ 72u_{11}u_{12}u_{20}y + 96u_{11}u_{12}u_{30}xy + 36u_{11}u_{12}x^2 - 12u_{11}u_{20}^2u_{40}x\\
&+ 24u_{11}u_{20}^2u_{41}xy - 108u_{11}u_{20}u_{31}x^2 - 384u_{11}u_{21}^2xy - 48u_{11}u_{21}u_{30}^2xy\\
&+ 24u_{12}u_{20}^2u_{30}y + 24u_{12}u_{20}u_{30}x^2 + u_{20}^4u_{40} - u_{20}^4u_{41}y\\
&- 4u_{20}^3u_{21}u_{40}y + 32u_{20}^3u_{21} + 4u_{20}^3u_{22}y + 4u_{20}^3u_{30}^2 - 8u_{20}^3u_{30}u_{31}y\\
&- 2u_{20}^3u_{41}x^2 - 96u_{20}^2u_{21}^2y - 12u_{20}^2u_{21}u_{30}^2y\\
&- 6u_{20}^2u_{21}u_{40}x^2 + 6u_{20}^2u_{22}x^2 - 12u_{20}^2u_{30}u_{31}x^2 - 96u_{20}u_{21}^2x^2\\
&- 12u_{20}u_{21}u_{30}^2x^2))/4
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(118) = {}&(\Omega_{yy}u_{30}(4Q_{2,5}x + 24Q_{3,4} - 24Q_{0,7}u_{10}y - 48Q_{0,7}u_{20}xy + 24Q_{0,9}xy\\
&- 36Q_{1,3}u_{10} + 24Q_{1,3}u_{11}y + 12Q_{1,3}u_{20}x + 72Q_{1,6}u_{20}y + 36Q_{1,6}x^2 + 12Q_{1,8}y\\
&+ 48uu_{01} + 96uu_{11}x + 24uu_{20}^2 - 12u_{01}^2y + 8u_{01}u_{10}x - 96u_{01}u_{11}xy\\
&- 48u_{01}u_{20}u_{30}xy - 12u_{01}u_{20}x^2 - 36u_{10}^2u_{20} - 24u_{10}u_{11}x^2 + 12u_{10}u_{20}^2x\\
&+ 24u_{11}u_{20}^2xy - u_{20}^4y - 2u_{20}^3x^2) + 2\Omega_{xy}u_{21}(-4Q_{2,5}x - 24Q_{3,4} + 24Q_{0,7}u_{10}y\\
&+ 48Q_{0,7}u_{20}xy - 24Q_{0,9}xy + 36Q_{1,3}u_{10} - 24Q_{1,3}u_{11}y - 12Q_{1,3}u_{20}x\\
&- 72Q_{1,6}u_{20}y - 36Q_{1,6}x^2 - 12Q_{1,8}y - 48uu_{01} - 96uu_{11}x - 24uu_{20}^2 + 12u_{01}^2y
\end{aligned}
$$

$$
\begin{aligned}
&- 8u_{01}u_{10}x + 96u_{01}u_{11}xy + 48u_{01}u_{20}u_{30}xy + 12u_{01}u_{20}x^2 + 36u_{10}^2u_{20}\\
&+ 24u_{10}u_{11}x^2 - 12u_{10}u_{20}^2x - 24u_{11}u_{20}^2xy + u_{20}^4y + 2u_{20}^3x^2)\\
&+ \Omega_{xx}u_{12}(4Q_{2,5}x + 24Q_{3,4} - 24Q_{0,7}u_{10}y - 48Q_{0,7}u_{20}xy + 24Q_{0,9}xy\\
&- 36Q_{1,3}u_{10} + 24Q_{1,3}u_{11}y + 12Q_{1,3}u_{20}x + 72Q_{1,6}u_{20}y + 36Q_{1,6}x^2 + 12Q_{1,8}y\\
&+ 48uu_{01} + 96uu_{11}x + 24uu_{20}^2 - 12u_{01}^2y + 8u_{01}u_{10}x - 96u_{01}u_{11}xy\\
&- 48u_{01}u_{20}u_{30}xy - 12u_{01}u_{20}x^2 - 36u_{10}^2u_{20} - 24u_{10}u_{11}x^2 + 12u_{10}u_{20}^2x\\
&+ 24u_{11}u_{20}^2xy - u_{20}^4y - 2u_{20}^3x^2) + 2\Omega_y(12Q_{2,5}u_{21} + 4Q_{2,5}u_{31}x + 24Q_{3,4}u_{31}\\
&- 24Q_{0,7}u_{10}u_{31}y - 48Q_{0,7}u_{20}u_{31}xy - 48Q_{0,7}u_{21}u_{30}xy + 36Q_{0,7}u_{21}x^2\\
&+ 36Q_{0,9}u_{21}y + 24Q_{0,9}u_{31}xy - 36Q_{1,3}u_{10}u_{31} + 24Q_{1,3}u_{11}u_{31}y\\
&+ 12Q_{1,3}u_{20}u_{31}x + 24Q_{1,3}u_{21}^2y + 12Q_{1,3}u_{21}u_{30}x + 72Q_{1,6}u_{20}u_{31}y\\
&+ 72Q_{1,6}u_{21}u_{30}y + 36Q_{1,6}u_{31}x^2 + 12Q_{1,8}u_{31}y + 48uu_{01}u_{31} + 144uu_{11}u_{21}\\
&+ 96uu_{11}u_{31}x + 24uu_{20}^2u_{31} + 48uu_{20}u_{21}u_{30} + 96uu_{21}^2x\\
&- 12u_{01}^2u_{31}y - 48u_{01}u_{10}u_{21} + 8u_{01}u_{10}u_{31}x - 72u_{01}u_{11}u_{21}y\\
&- 96u_{01}u_{11}u_{31}xy - 24u_{01}u_{20}u_{21}u_{30}y - 48u_{01}u_{20}u_{30}u_{31}xy - 12u_{01}u_{20}u_{31}x^2\\
&- 96u_{01}u_{21}^2xy - 48u_{01}u_{21}u_{30}^2xy + 24u_{01}u_{21}u_{30}x^2 - 36u_{10}^2u_{20}u_{31}\\
&- 36u_{10}^2u_{21}u_{30} + 72u_{10}u_{11}u_{21}x - 24u_{10}u_{11}u_{31}x^2 + 12u_{10}u_{20}^2u_{31}x\\
&+ 24u_{10}u_{20}u_{21}u_{30}x - 24u_{10}u_{21}^2x^2 - 72u_{11}^2u_{21}xy + 24u_{11}u_{20}^2u_{31}xy\\
&- 36u_{11}u_{20}u_{21}x^2 - u_{20}^4u_{31}y - 4u_{20}^3u_{21}u_{30}y - 2u_{20}^3u_{31}x^2 - 6u_{20}^2u_{21}u_{30}x^2)\\
&+ \Omega_x(-12Q_{2,5}u_{12} - 4Q_{2,5}u_{22}x - 24Q_{3,4}u_{22} + 24Q_{0,7}u_{10}u_{22}y\\
&+ 48Q_{0,7}u_{12}u_{30}xy - 36Q_{0,7}u_{12}x^2 + 48Q_{0,7}u_{20}u_{22}xy - 36Q_{0,9}u_{12}y\\
&- 24Q_{0,9}u_{22}xy + 36Q_{1,3}u_{10}u_{22} - 24Q_{1,3}u_{11}u_{22}y - 24Q_{1,3}u_{12}u_{21}y\\
&- 12Q_{1,3}u_{12}u_{30}x - 12Q_{1,3}u_{20}u_{22}x - 72Q_{1,6}u_{12}u_{30}y - 72Q_{1,6}u_{20}u_{22}y\\
&- 36Q_{1,6}u_{22}x^2 - 12Q_{1,8}u_{22}y - 48uu_{01}u_{22} - 144uu_{11}u_{12} - 96uu_{11}u_{22}x\\
&- 48uu_{12}u_{20}u_{30} - 96uu_{12}u_{21}x - 24uu_{20}^2u_{22} + 12u_{01}^2u_{22}y\\
&+ 48u_{01}u_{10}u_{12} - 8u_{01}u_{10}u_{22}x + 72u_{01}u_{11}u_{12}y + 96u_{01}u_{11}u_{22}xy\\
&+ 24u_{01}u_{12}u_{20}u_{30}y + 96u_{01}u_{12}u_{21}xy + 48u_{01}u_{12}u_{30}^2xy - 24u_{01}u_{12}u_{30}x^2\\
&+ 48u_{01}u_{20}u_{22}u_{30}xy + 12u_{01}u_{20}u_{22}x^2 + 36u_{10}^2u_{12}u_{30} + 36u_{10}^2u_{20}u_{22}\\
&- 72u_{10}u_{11}u_{12}x + 24u_{10}u_{11}u_{22}x^2 - 24u_{10}u_{12}u_{20}u_{30}x + 24u_{10}u_{12}u_{21}x^2\\
&- 12u_{10}u_{20}^2u_{22}x + 72u_{11}^2u_{12}xy + 36u_{11}u_{12}u_{20}x^2 - 24u_{11}u_{20}^2u_{22}xy\\
&+ 4u_{12}u_{20}^3u_{30}y + 6u_{12}u_{20}^2u_{30}x^2 + u_{20}^4u_{22}y + 2u_{20}^3u_{22}x^2)\\
&+ \Omega(24Q_{2,5}u_{22} + 4Q_{2,5}u_{32}x + 24Q_{3,4}u_{32} - 24Q_{0,7}u_{10}u_{32}y + 24Q_{0,7}u_{12}u_{30}y\\
&- 48Q_{0,7}u_{12}u_{40}xy + 72Q_{0,7}u_{12}x - 48Q_{0,7}u_{20}u_{32}xy - 96Q_{0,7}u_{22}u_{30}xy\\
&+ 72Q_{0,7}u_{22}x^2 + 72Q_{0,9}u_{22}y + 24Q_{0,9}u_{32}xy - 36Q_{1,3}u_{10}u_{32} + 24Q_{1,3}u_{11}u_{32}y\\
&+ 12Q_{1,3}u_{12}u_{30} + 24Q_{1,3}u_{12}u_{31}y + 12Q_{1,3}u_{12}u_{40}x + 12Q_{1,3}u_{20}u_{32}x\\
&+ 48Q_{1,3}u_{21}u_{22}y + 24Q_{1,3}u_{22}u_{30}x + 72Q_{1,6}u_{12}u_{40}y - 216Q_{1,6}u_{12}\\
&+ 72Q_{1,6}u_{20}u_{32}y + 144Q_{1,6}u_{22}u_{30}y + 36Q_{1,6}u_{32}x^2 + 12Q_{1,8}u_{32}y + 48uu_{01}u_{32}\\
&+ 288uu_{11}u_{22} + 96uu_{11}u_{32}x + 48uu_{12}u_{20}u_{40} + 240uu_{12}u_{21} + 48uu_{12}u_{30}^2\\
&+ 96uu_{12}u_{31}x + 24uu_{20}^2u_{32} + 96uu_{20}u_{22}u_{30} + 192uu_{21}u_{22}x - 12u_{01}^2u_{32}y\\
&- 96u_{01}u_{10}u_{22} + 8u_{01}u_{10}u_{32}x - 144u_{01}u_{11}u_{22}y - 96u_{01}u_{11}u_{32}xy\\
&- 24u_{01}u_{12}u_{20}u_{40}y - 72u_{01}u_{12}u_{20} - 120u_{01}u_{12}u_{21}y - 48u_{01}u_{12}u_{30}u_{40}xy\\
&+ 72u_{01}u_{12}u_{30}x - 96u_{01}u_{12}u_{31}xy - 12u_{01}u_{12}u_{40}x^2 - 48u_{01}u_{20}u_{22}u_{30}y
\end{aligned}
$$

$$
\begin{aligned}
&- 48u_{01}u_{20}u_{30}u_{32}xy - 12u_{01}u_{20}u_{32}x^2 - 192u_{01}u_{21}u_{22}xy - 96u_{01}u_{22}u_{30}^2xy\\
&+ 48u_{01}u_{22}u_{30}x^2 + 72u_{02}u_{12}u_{20}y - 48u_{02}u_{12}u_{30}xy\\
&+ 36u_{02}u_{12}x^2 - 36u_{10}^2u_{12}u_{40} - 36u_{10}^2u_{20}u_{32} - 72u_{10}^2u_{22}u_{30} + 216u_{10}u_{11}u_{12}\\
&+ 144u_{10}u_{11}u_{22}x - 24u_{10}u_{11}u_{32}x^2 + 24u_{10}u_{12}u_{20}u_{40}x + 120u_{10}u_{12}u_{21}x\\
&+ 24u_{10}u_{12}u_{30}^2x - 24u_{10}u_{12}u_{31}x^2 + 12u_{10}u_{20}^2u_{32}x + 48u_{10}u_{20}u_{22}u_{30}x\\
&- 48u_{10}u_{21}u_{22}x^2 - 108u_{11}^2u_{12}y - 144u_{11}^2u_{22}xy - 24u_{11}u_{12}u_{20}u_{30}y\\
&- 240u_{11}u_{12}u_{21}xy - 48u_{11}u_{12}u_{30}^2xy - 12u_{11}u_{12}u_{30}x^2\\
&+ 24u_{11}u_{20}^2u_{32}xy - 72u_{11}u_{20}u_{22}x^2 - 4u_{12}u_{20}^3u_{40}y + 12u_{12}u_{20}^3 - 60u_{12}u_{20}^2u_{21}y\\
&- 12u_{12}u_{20}^2u_{30}^2y - 6u_{12}u_{20}^2u_{40}x^2 - 60u_{12}u_{20}u_{21}x^2 - 12u_{12}u_{20}u_{30}^2x^2 - u_{20}^4u_{32}y\\
&- 8u_{20}^3u_{22}u_{30}y - 2u_{20}^3u_{32}x^2 - 12u_{20}^2u_{22}u_{30}x^2))/4
\end{aligned}
$$

$$
\begin{aligned}
D_x\Omega(119) = {}&(\Omega_{yy}(-2u_{10}y + 2u_{11}y^2 + 2u_{20}xy - x^3)\\
&+ \Omega_y(2u_{10}u_{40}y + 2u_{10} - 2u_{11}u_{40}y^2 - 2u_{11}y - 2u_{12}y^2 - 2u_{20}u_{40}xy - 2u_{20}x\\
&- 2u_{21}u_{30}y^2 - 2u_{21}xy - 2u_{30}^2xy + 3u_{30}x^2 + u_{40}x^3)\\
&+ \Omega(-2u_{10}u_{40} + 2u_{10}u_{41}y + 2u_{11}u_{40}y - 2u_{11}u_{41}y^2 + 6u_{12}y + 2u_{20}u_{40}x\\
&- 2u_{20}u_{41}xy - 6u_{21}u_{31}y^2 - 2u_{21}u_{40}xy + 16u_{21}x + 2u_{22}xy + 2u_{30}^2x - 4u_{30}u_{31}xy\\
&+ 9u_{31}x^2 + u_{41}x^3))/2
\end{aligned}
$$

$$
\begin{aligned}
D_y\Omega(119) = {}&(\Omega_{yy}u_{30}(2u_{10}y - 2u_{11}y^2 - 2u_{20}xy + x^3)\\
&+ 2\Omega_{xy}u_{21}(-2u_{10}y + 2u_{11}y^2 + 2u_{20}xy - x^3)\\
&+ \Omega_{xx}u_{12}(2u_{10}y - 2u_{11}y^2 - 2u_{20}xy + x^3)\\
&+ 2\Omega_y(2u_{10}u_{31}y - 2u_{11}u_{31}y^2 - 2u_{20}u_{31}xy - 2u_{21}^2y^2\\
&- 2u_{21}u_{30}xy + 3u_{21}x^2 + u_{31}x^3)\\
&+ \Omega_x(-2u_{10}u_{22}y + 2u_{11}u_{22}y^2 + 2u_{12}u_{21}y^2 + 2u_{12}u_{30}xy - 3u_{12}x^2\\
&+ 2u_{20}u_{22}xy - u_{22}x^3)\\
&+ \Omega(2u_{10}u_{32}y - 2u_{11}u_{32}y^2 - 2u_{12}u_{30}y - 2u_{12}u_{31}y^2 - 2u_{12}u_{40}xy + 6u_{12}x\\
&- 2u_{20}u_{32}xy - 4u_{21}u_{22}y^2 - 4u_{22}u_{30}xy + 6u_{22}x^2 + u_{32}x^3))/2
\end{aligned}
$$

The x and y derivatives given above fit perfectly in the canonical presentation and are obtained from functions G_*.

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